**The Algebra of Tax Burden Shifts
from Assessment Limitations**

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Abstract

Although assessment limits appear to be an attractive policy option in areas with rapidly growing market values because they keep a homeowner's property tax from rising rapidly without placing restrictions on expenditure growth, the policies often have unintended distributional effects. A stylized model demonstrates that taxes *must* rise for some groups in order to provide tax relief to others. The model confirms that an apparently surprising result is a logical necessity of assessment limits: taxes can *increase* for groups who appear to be enjoying tax relief. The extent of the tax increase is higher (1) the greater the appreciation rate for other favored properties and (2) the higher the percentage of favored properties with higher appreciation rates.

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The Algebra of Tax Burden Shifts from Assessment Limitations

1. Introduction

This paper is a companion to our Lincoln Institute *Land Lines* article "Surprise! An unintended consequence of assessment limitations" (Dye and McMillen, 2007). In that shorter paper we present simple numerical examples to show that some property owners seemingly targeted to benefit from a limitation on assessments may be harmed instead. Here we present a more general algebraic model of the tax shift to from property tax assessment growth limitations and make the same point. The reader is also directed to Dornfest (2005) for an excellent, but different, way of presenting many of the same results.

Rapidly rising housing prices have produced property tax revolts in many areas where assessments respond to market values. Youngman and Malme (2005) summarize various types of policy responses to property tax volatility, including direct limits on tax rates as well as limits on revenue and expenditure increases. According to a recent study by Anderson (2006), all but 5 of the 48 continental United States have some form of explicit limits on property taxes. Assessment limits typically take the form of a restriction on the annual percentage increase in a property's assessed value, the extreme form being a freeze—no increase for the duration of a homeowner's residence. Assessment limits are popular because they do not directly restrict a jurisdiction's ability to raise revenue for desired government services, while providing some insurance that long-term homeowners' property taxes will grow to exceed their ability to pay. Anderson (2006) reports that 20 states had some form of assessment limits in 2006.

The attraction of assessment limits is seemingly a paradox: the limits hold down property taxes without restricting expenditure growth. The resolution of the paradox is found in the distributional effects of assessment limits. All states explicitly exempt some classes of properties from the limits. Single-family homes are the most favored category, while assessment increases are typically left unrestricted for commercial and manufacturing properties. Not surprisingly, studies such as Dye, Merriman, and McMillen (2006) and Minnesota Department of Revenue (2007) have found that assessment limitation measures have transferred the tax burden from favored property classes to properties that are exempt from the limits.

The distributional effects of assessment limits are not always obvious beforehand. Even within the class of favored properties, assessment limits transfer taxes from areas where sales prices are appreciating rapidly to areas with low growth rates. More surprisingly, taxes may actually increase for homeowners who appear to be benefiting from assessment limits. For example, a property whose assessment would have increased by 8% in a market with a 7% cap on assessment increases will certainly appear to have been a beneficiary of the limitation measure. Yet its taxes may actually be higher than they would have been without assessment limits. This ironic outcome can come about if many other properties in the jurisdiction are appreciating much more rapidly. The tax benefits enjoyed by rapidly appreciating properties lead to a higher tax *rate*, which can, in turn, lead to higher tax payments even for properties that appear to be enjoying tax relief.

In this paper, we use a stylized model to show how an assessment limitation measure can produce these ironic results. Simple algebra shows that tax payments rise for properties with appreciation rates below the limits. Some algebra also shows that taxes can increase significantly even for properties with appreciation rates above the limits. The extent of the tax increase is higher (1) the greater the appreciation rate for other favored properties and (2) the higher the percentage of favored properties with higher appreciation rates. Naturally, actual policies are more complicated than the stylized model developed here. But the model accounts for the important features of existing policies and shows that these apparently surprising results are actually an inherent feature of any tax limitation measure that attempts to provide tax relief without curbing expenditures.

2. Assessment Limit Algebra – Multiple Groups with Different Appreciation Rates

The stylized jurisdiction has a simple proportional property tax applied to each of N properties. All properties have the same value, V, in the initial period. The tax system has no homestead exemption or other complications that would detract from the basic point of the model. The total tax levy is T and the tax rate is $t = T/B$, where B is the tax base. Properties are assumed to be assessed accurately at market value, so $B = NV$.

Although all properties have the same value in the initial period, they are appreciating at different rates. Properties are distinguished by their appreciation rate, g. The appreciation rate for property type j is g_j, with $j=1,\ldots,J$ and $J \leq N$. The number of properties of type j is $p_j N$, where p_j is the proportion of type-j properties and $\sum_{j=1}^{J} p_j = 1$. With no assessment cap, the tax base in

the second period is $B_u = NV \sum_{j=1}^{J} p_j (1+g_j) = = B\left(1 + \sum_{j=1}^{J} p_j g_j \right)$, where the subscript u denotes

the unrestricted second-period tax base. The second period tax rate is then $t_u = T/B_u$. With T constant across the periods, $t_u < t$ since $B_u > B$.

The assessment cap constrains the growth in assessed value between periods to the rate g_r. When the actual appreciation rate is higher than the capped rate, the assessed value in the second period is $V(1+g_r)$ rather than $V(1+g_j)$. The assessed value for property type j in the second period can be written as $V\left(1+g_j^*\right)$, where $g_j^* = \min\left(g_j, g_r\right)$. The restricted second-

period tax base is $B_r = NV \sum_{j=1}^{J} p_j \left(1+g_j^*\right) = B\left(1 + \sum_{j=1}^{J} p_j g_j^* \right)$, and the tax rate is $t_r = T/B_r$. The

assumptions for appreciation rates imply that $B < B_r < B_u$ and $t > t_r > t_u$. Higher tax rates are a direct result of an assessment cap. Higher rates are necessary in order to collect the same amount revenue from a reduced tax base.

The next step is to compare tax payments across the different property types. The second-period tax payment for a property from group i is $T_{ui} = t_u V (1 + g_i)$ in the absence of an assessment cap, and it is $T_{ri} = t_r V (1 + g_i^*)$ with the cap. The proportionate change in tax rates is

$D_i = \ln T_{ri} - \ln T_{ui} = \ln t_r - \ln t_u + \ln (1 + g_i^*) - \ln (1 + g_i)$. Substituting for t_r and t_u, we have

$$D_i = -(\ln B_r - \ln B_u) + \ln (1 + g_i^*) - \ln (1 + g_i). \qquad (1)$$

With small growth rates, this expression can be simplified using the following approximations:

$\ln B_u \approx \ln B + \sum_{j=1}^{J} p_j g_j$, $\ln B_r \approx \ln B + \sum_{j=1}^{J} p_j g_j^*$, $\ln (1 + g_i^*) \approx g_i^*$, and $\ln (1 + g_i) \approx g_i$. With these

approximations, we have $D_i \approx -\sum_{j=1}^{J} p_j (g_j^* - g_j) + (g_i^* - g_i)$, or

$$D_i = (1 - p_i)(g_i^* - g_i) - \sum_{j \neq i} p_j (g_j^* - g_j) \qquad (2)$$

This equation directly compares the tax payment for property type i after the assessment cap to the payment in the absence of a cap.

Equation (2) shows that the proportionate change in tax payments depends on relative appreciation rates and the number of properties of each type. Suppose that property type i is the only group with appreciation rates above the cap. Thus, $g_i^* < g_i$ and $g_j^* = g_j$ for $j \neq i$. In this case, $D_i = (1 - p_i)(g_i^* - g_i) < 0$, i.e., the cap leads to a lower tax payment for each property in group i. The reduction in the tax payment is larger (1) the greater the difference between the actual appreciation rate and the capped rate, and (2) the lower the percentage of properties that benefit from the cap. The reason for the second result is that the reduction in the tax base leads to a higher tax *rate*. This increase in the tax rate reduces but does not eliminate the benefits of the cap for the properties with appreciation rates exceeding g_i^*. The higher tax rate means that properties with appreciation rates below g_i^* have higher taxes than they would have paid in the absence of the cap: $D_j = -p_i (g_i^* - g_i) > 0$. The increased tax payment for properties with low appreciation rates is higher the higher the percentage of capped properties and the greater the difference between the capped and actual appreciation rates for capped properties.

So far these results are not surprising: capped properties pay less under an assessment cap while uncapped properties pay more. This result holds whether the uncapped properties are simply not eligible for the tax or have appreciation rates falling below the assessment cap rate. The best situation for a taxpayer is to be one of only a handful of capped properties. The worst situation is to be one of a small number of uncapped properties in a market where values are appreciating rapidly in the favored property class.

The results are more surprising when we compare groups that appear to be gaining from the assessment cap. To simplify matters without affecting the results, assume that only two groups benefit from the tax. The proportionate change in tax payments induced by the cap is

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$D_1 = (1 - p_1)(g_1^* - g_1) - p_2(g_2^* - g_2)$ for group 1. With both g_1 and g_2 greater than the capped value, this expression may be either positive or negative. The tax payment declines

if $g_1^* - g_1 < \dfrac{p_2}{1 - p_1}(g_2^* - g_2)$, i.e., if group 1's appreciation rate is relatively far above the capped

value and it comprises a relatively small proportion of the total number of capped properties. In contrast, the tax payment will increase if the inequality is reversed, i.e., if group 1's appreciation rate is close to the capped value and it comprises a relatively large proportion of the total number of capped properties.

The analysis suggests that at least three groups must be considered when calculating tax burden shifts caused by an assessment cap. The first group—uncapped or very slowly appreciating properties—clearly pays more after an assessment cap. While these properties are assessed at the same value with or without the cap, the higher tax *rate* increases their payments. The second group—properties with high appreciation rates—clearly gains by lower assessments even though these properties too are subject to a higher rate than before. But a third group that appears to gain from the assessment cap may actually pay higher taxes after the cap. This group comprises properties with assessment rates close to (but in excess of) the cap in jurisdiction with a high proportion of properties with even higher appreciation rates. In other words, when the cap is 5% it is good to be one of 100 homes with appreciation rates of 20% in a jurisdiction with 1000 homes that are appreciating at 6%. It is bad to be one of 100 homes with appreciation rates of 6% when 1000 homes are appreciating at 20%.

3. A Simplification of the Model – Three Groups with an Exempt Sector

The same results can be shown in a simplification of the model with just shares of the total tax base rather than counts of individual properties. The stylized jurisdiction has three types of property: property not eligible for the cap (N), cap-eligible property with a low appreciation rate (L), and cap-eligible property with a high appreciation rate (H). The initial shares of total market value are p_N, p_L, and p_H respectively. Properties are accurately assessed at market value, which aggregates to the tax base (B). The total tax levy is T and the tax rate is $t = T / B$. Property groups are distinguished by their appreciation rates—g_N, g_L, and g_H.

With no assessment cap, the unrestricted tax base in the second period is
$B_u = B[p_N(1 + g_N) + p_L(1 + p_L) + p_H(1 + p_H)]$. With T constant across periods, the unrestricted second period tax rate is then $t_u = T / B_u$,

The assessment cap restricts the growth in assessed value between periods for properties in covered groups L and H to no greater than the rate g_r. Thus the assessment growth rates are
$g_L^* = \min(g_L, g_r)$ and $g_H^* = \min(g_H, g_r)$ respectively. The restricted second-period tax base

is $B_r = B[p_N(1 + g_N) + p_L(1 + p_L^*) + p_H(1 + p_H^*)]$, and the tax rate is $t_r = T / B_r$. The assumptions for appreciation rates imply that $B < B_r < B_u$ and $t > t_r > t_u$.

The next step is to compare tax payments across the different property types. The second-period tax payment for a property from group i is $T_{ui} = t_u B p_i (1 + g_i)$ in the absence of an assessment

cap, and it is $T_{ri} = t_r B p_i \left(1 + g_i^*\right)$ with the cap. The proportionate change in tax rates is

$D_i = \ln T_{ri} - \ln T_{ui} = \ln t_r - \ln t_u + \ln\left(1 + g_i^*\right) - \ln\left(1 + g_i\right)$. Substituting for t_r and t_u, we have

$$D_i = -\left(\ln B_r - \ln B_u\right) + \ln\left(1 + g_i^*\right) - \ln\left(1 + g_i\right). \qquad (1')$$

With small growth rates, this expression can be approximated as

$$D_i = (1 - p_i)\left(g_i^* - g_i\right) - \sum_{j \neq i} p_j \left(g_j^* - g_j\right) \qquad (2')$$

This equation directly compares the tax payment for property type i after the assessment cap to the payment in the absence of a cap. We use this expression for the numerical simulations of D_N, D_L, and D_H from different values of the other parameters presented in Figures 1-4.

Equation (2) can be further simplified for each of the three types of property and by assuming that $g_H > g_L > g_r$. Here, N means not capped, H means high growth, L means lower growth, and the assessment cap is binding on both H and L.

$$D_N = p_L \left(g_L - g_r\right) + p_H \left(g_H - g_r\right) \qquad (2'N)$$

$$D_H = p_N \left(g_r - g_H\right) + p_L \left(g_L - g_H\right) \qquad (2'H)$$

$$D_L = p_N \left(g_r - g_L\right) + p_H \left(g_H - g_L\right) \qquad (2'L)$$

Given the assumed ordering of the growth rates, both terms in equation (2'N) are positive—taxes will go up for the not-capped group when the other groups have their assessments lowered to the restricted rate. The larger are the shares for the favored groups (p_L or p_H), the larger will be the shift in burden to the not-capped property owners.

Given the assumed ordering of the growth rates, both terms in equation (2'H) are negative—showing the other obvious result that high-growth properties have tax payments lowered by their relative advantage over both the other groups. Their advantage is greater the larger are the shares of the other two groups to which they can pass the burden.

For the covered but low-growth properties the net impact is ambiguous. The first term in equation (2'L) is negative—they have their assessments reduced and thus gain relative to the not-capped group and the larger is that group the more they gain. But, the second term is positive—they pay higher taxes to the extent they grow less than the high-growth group and the larger is that group the more they lose.

4. An Illustration and Examples

There are three groups of properties: group N properties are not covered by the assessment limitation; group H properties are covered and have high appreciation; group L properties are covered but have relatively lower rates of appreciation. With the assumption that tax rates (t) will adjust to maintain a fixed tax levy (T), the model in the previous section demonstrates that the distributional effects are a function of the following variables: the initial percentage shares of the three types of property in the total tax base – p_N, p_H, and p_L; the growth rates of property values in each of these groups – g_N, g_H, and g_L; and appreciation cap applied in the restricted

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case – g_r. We perform a number of simulations with different values of the parameters and report the percentage difference between the unrestricted and capped tax payments for each group.

Figure 1 shows the percentage difference between the unrestricted and capped tax payments in the second period when the appreciation rate for group L properties is at or below the cap. The specific assumptions are: the assessment cap restriction is set at $g_r = 5\%$; properties in group N represent $p_N = 20\%$ of the initial tax base and are growing at $g_N = 5\%$; properties in covered group L represent $p_L = 20\%$ of the initial tax base and are growing at $g_L = 5\%$; the appreciation rates are allowed to vary from $g_H = 0\text{-}20\%$ for covered properties in group H. As expected, groups N and L begin to pay higher taxes under the cap once g_H rises above the 5% cap. When $g_H = 20\%$, the tax rate under the cap is 3.81% rather than the 3.51% without the cap. Property tax payments are 8.57% higher for groups N and L under the cap than they would have been without the cap. Property tax payments are 5.00% lower for group H under the cap. But tax relief for group H comes at the expense of higher payments for groups N and L.

Figure 1:
Percentage Difference in Tax Bills due to Assessment Cap
With Group L at (or below) the Cap Rate and a Large Group H
By Cap-Status Group for Different Growth Rates for Group H
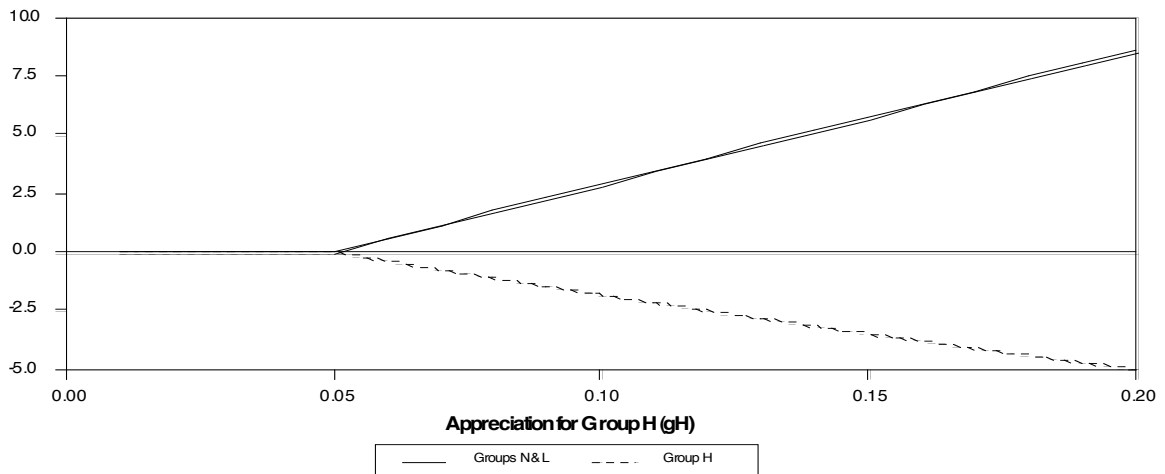($g_r = .05$, $g_N = .05$, $g_L = .05$, $p_N = .2$, $p_L = .2$, $p_H = .6$)

Figure 2 shows the percentage difference between the unrestricted and capped tax payments when the appreciation rate for group L properties is above the cap. For this example, $g_L = 6\%$ while all other parameters are as before. In this case, group L is the only set of properties receiving benefits from the assessment cap as long as $g_H \leq 5\%$. As expected, the percentage difference between the unrestricted and capped tax payments is negative for group L when $g_H \leq 5\%$, i.e., this group enjoys tax savings. It follows that tax payments are higher for the other two groups than would have been the case without the assessment cap when $g_H \leq 5\%$. The situation begins to change as group H begins to gain from the assessment cap, i.e., as g_H rises above 5%. By the time g_H rises to only 7%, group L no longer benefits from the assessment cap. With $p_H =$

60%, group H is a large proportion of the total set of taxpayers. The assessment cap leads to a higher tax rate that more than offsets the gains group L receives from the assessment cap. Group N receives no benefits from the assessment cap, and has higher tax payments under the cap than without the cap for all values of g_H.

Figure 2:
Percentage Difference in Tax Bills due to Assessment Cap
With Group L Above the Cap Rate and a Large Group H
By Cap-Status Group for Different Growth Rates for Group H
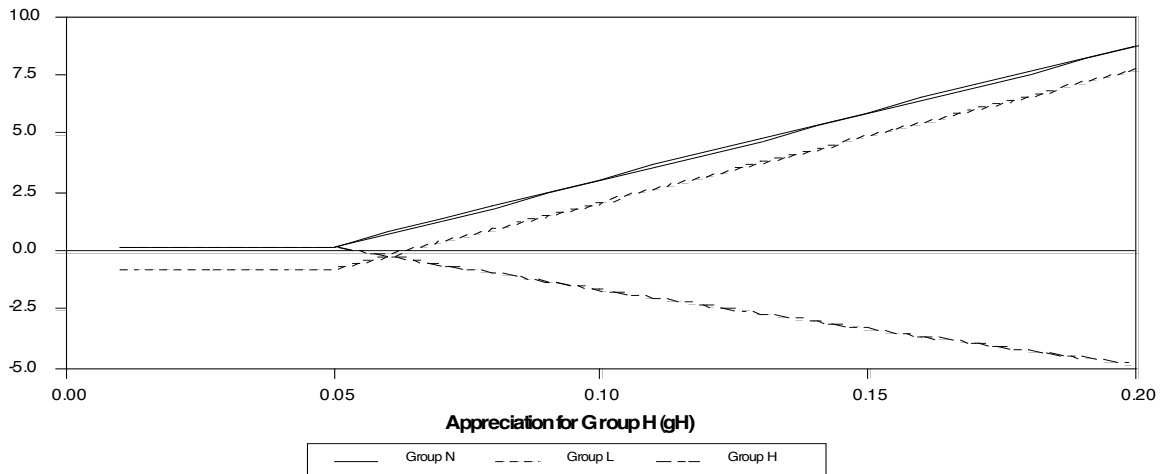$(g_r = .05, g_N = .05, g_L = .06, p_N = .2, p_L = .2, p_H = .6)$

Figure 3 shows the effects of a change in the group size and makes the simple, but important, point that the larger is the uncapped group N, the smaller is the *percentage* increase in tax rates needed to pay for tax relief for favored group H. The assumptions are identical to Figure 1, except that the shares for groups N and H are reversed ($p_N = 60\%$ and $p_H = 20\%$). Compare these results to Figure 1 for value of $g_H = 20\%$ (the right-hand side of the figures). With the larger uncapped group on which to shift the burden, in Figure 3 the property tax payments are 10.00% lower for group H (as opposed to 5.00% in Figure 1) and only 2.86% higher for groups N and L (as opposed to 8.57% in Figure 1).

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Figure 3:
Percentage Difference in Tax Bills due to Assessment Cap
With Group L at (or below) the Cap Rate and a *Small* Group H

By Cap-Status Group for Different Growth Rates for Group H
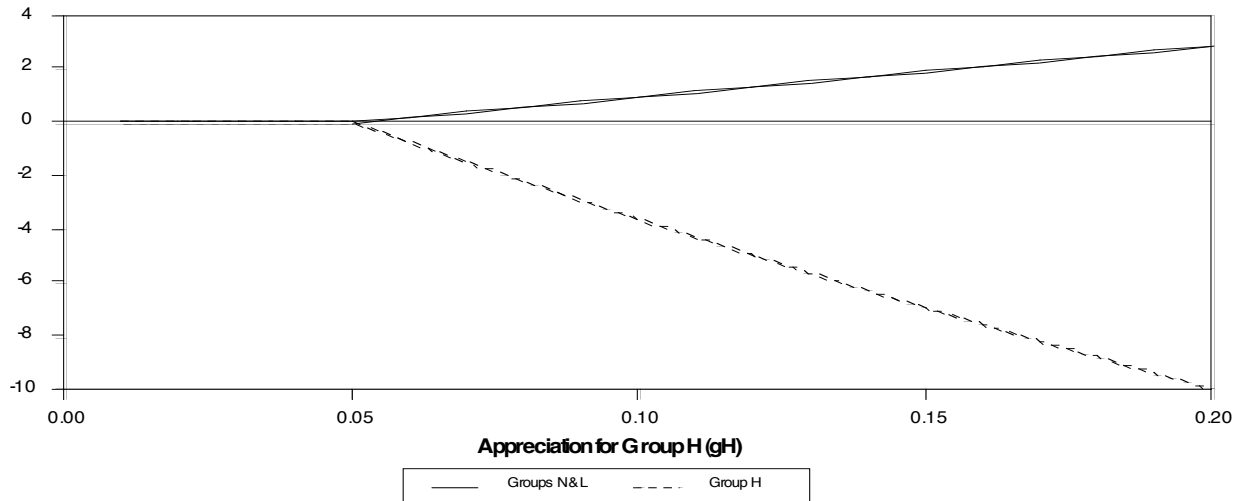$(g_r = .05, g_N = .05, g_L = .05, p_N = .6, p_L = .2, p_H = .2)$

Figure 4 examines the effect of the relative sizes of group L and H where the appreciation rate for L is above the cap rate. The assumptions are identical to Figure 2, except that the shares for groups L and H are reversed ($p_L = 60\%$ and $p_H = 20\%$). In this example, group L gains more from the assessment cap when $g_H \leq 5\%$. Group L's tax payments do not increase as rapidly with g_H as before, while each property in group H enjoys a greater percentage reduction in its payments. Since group H is relatively small, it can enjoy large reductions in tax payments before tax rates increase significantly.

Figure 4:
Percentage Difference in Tax Bills due to Assessment Cap
With a *Large* Group L Above the Cap Rate and a *Small* Group H
By Cap-Status Group for Different Growth Rates for Group H
(g_r = .05, g_N = .05, g_L = .06, p_N = .2, p_L = .6, p_H = .2
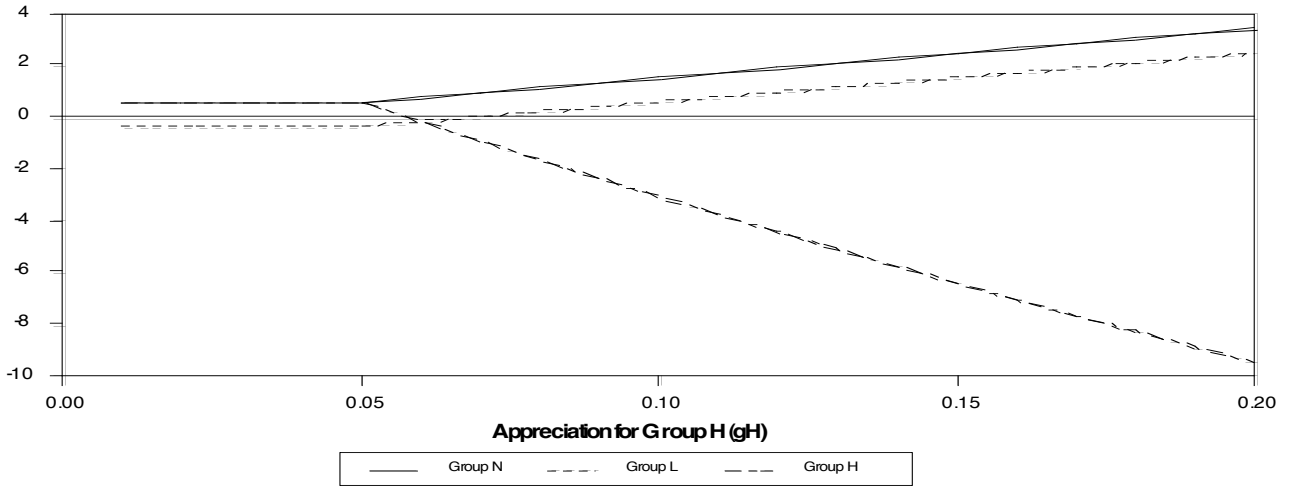
5. Conclusion

Assessment caps appear to be an attractive policy option because they keep a homeowner's property tax from rising rapidly without placing restrictions on expenditure growth. Caps succeed in keeping property taxes down in rapidly appreciating areas by transferring tax payments to exempt sectors and homes in areas with low appreciation rates. To keep revenues from falling, tax rates must rise. Simple algebra shows that taxes *must* rise for some groups in order to provide tax relief to others.

Many observers are surprised to find that taxes may actually rise for groups who appear to be enjoying tax relief under an assessment cap. If a large proportion of revenue in a jurisdiction comes from properties with high appreciation rates, taxes will be higher for properties with appreciation rates that are above but close to the cap. Homeowners with appreciation rates of 7% in a jurisdiction with a cap of 5% can pay more than they would otherwise if many properties are appreciating at much higher rates. The primary effect of an assessment cap is to shift tax burdens from favored to un-favored groups and from properties with high appreciation rates to those that are appreciating slowly.

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