How Differences in Property Taxes within Cities Affect Urban Sprawl

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Abstract

This article attempts a formal analysis of the connection between the differentiated property tax within urban areas and urban sprawl in U.S. cities. We first develop a theoretical model in which the city is duocentric, where the Central Business District (CBD) is located at the origin while the Suburban Business District (SBD hereafter) is at the other end of the city. We show that the ratio between the property tax in the suburbs and in the center has an ambiguous impact on the size of the city. We then test this model empirically to determine this sign by using a dataset of effective property tax rates we developed using GIS techniques for central cities and suburbs in 448 urbanized areas. The empirical analysis estimates a regression equation relating an urbanized area's size to the ratio of property tax rate in suburbs to the rate in central cities and other control variables such as population, income, agricultural rent, and transportation expenditure. Results from the empirical analysis suggest that a lower ratio between the property tax in the suburbs and in the center results in urban sprawl.

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1 Introduction

Urban sprawl is a pejorative term that connotes the undesirable features of contemporary urban development patterns. Such features include, for example, low density and separated land uses, automobile orientation, and unsightliness (Ewing, 1997; Downs, 1998; The Sierra Club, 1999). This urban sprawl development pattern has been criticized for consuming a large amount of land, demanding huge investment on transportation and facilities, worsening social inequality, threatening the environment and so forth.[1] The recent US 2000 census data provided considerable new evidence on the problem of urban sprawl. With these data, researchers have shown that urban areas continue to grow faster than their populations, causing urban densities to fall. These trends suggest that development patterns in the US are getting more unsustainable.

However, urban sprawl cannot be attributed to a single cause. In a recent study by Burchfield et al. (2006), ground water availability, temperate climate, rugged terrain, decentralized employment, early public transport infrastructure, uncertainty about metropolitan growth, and unincorporated land in the urban fringe are found to increase sprawl. In addition, the long-standing debate on land taxation and its virtues (George, 1879; Skaburskis and Tomalty, 1997) reveals that the property tax might be one of the potential causes of urban sprawl. The property tax can be viewed as a tax levied at equal rates on both the land and capital embodied in structures while, in a pure land tax, the tax on capital (i.e., improvements) is set to zero. The literature — for example, Arnott and MacKinnon, 1977; Case and Grant, 1991; Oates and Schwab, 1997; Mills, 1998; and Brueckner and Kim, 2003 — provides an abundance of arguments for how the property tax may influence land development.

Brueckner and Kim (2003) were the first to provide a theoretical analysis that incorporates a land market to investigate the connection between urban spatial expansion and the property tax. In their equilibrium analysis, they found two countervailing effects of the property tax on the spatial size of cities. On the one hand, the *improvement effect* refers to the impact of the property tax in lowering the equilibrium level of improvements chosen by the developer. The lower level of improvements per acre implies a reduction in the intensity of land development and this lower density associated with property tax appears to encourage urban sprawl. On the other hand, the *dwelling size effect* operates through the property tax's impact on the consumer's choice of dwelling sizes. As the tax on land and structures is partly shifted forward to

[1] For overviews on urban sprawl issues, see Brueckner (2000), Glaeser and Kahn (2004), Nechyba and Walsh (2004).

consumers, dwelling size decreases due to a higher cost of housing floor space. The reduction in dwelling size implies an increase in population density and thus, a decrease in the city's size or spatial extent. In Brueckner and Kim's full analysis, the net effect of the property tax on the spatial extent of a city is ambiguous. However, using a Constant Elasticity of Substitution (CES) utility function with an elasticity of substitution greater than or equal to one, Brueckner and Kim (2003) were able to show that the relationship between the property tax and urban sprawl is always negative.

Following this line of research, Song and Zenou (2006) develop a theoretical model in the same vein as Brueckner and Kim (2003) but with a log-linear utility function, which exhibits variable, rather than constant, elasticity of substitution but where the elasticity also exceeds one. The main feature of their utility function is that it has a zero income elasticity of housing demand but it allows us (contrary to the CES case) to have explicit closed-form solutions. They unambiguously show that increasing property taxes reduces the size of the city and thus, urban sprawl.

However, Song and Zenou's study is limited since it does not differentiate property tax rates within urbanized areas while the variation of tax rates between central cities and suburbs is a common feature in US metropolitan areas. Indeed, differentiated tax rates within urbanized areas can affect the spread of an urban region. The hypothesis is that if the central city has higher property tax rate than the suburbs, more developments would then occur in the suburbs. In other words, lower property tax rate in suburbs would induce developments and thus cause urban sprawl. This is an important issue and we attempt to examine it in the present paper. For that, we extend our previous research (Song and Zenou, 2006) by taking explicitly into account different property tax rates in different areas of the city. We develop a duocentric urban model where residents pay different property taxes depending where they reside. There are two main areas: the central part of the city and the suburbs. Firms/developers enter in the market in one of the areas and decide their level of improvement. We characterize the equilibrium in this city and show that the ratio between the property tax in the suburbs and in the center has an ambiguous impact on the size of the city. We give a condition under which this impact is negative.

We then test empirically the main result of our model. We collect data on the property effective tax rates from various taxing jurisdictions and develop a sample of effective tax rates for 448 urbanized areas in the U.S. For each urbanized area, we further divide the area between the central city and the suburbs. Results from the empirical analysis suggest that a lower ratio between the property tax in the suburbs and in the center results in urban sprawl.

The present paper also contributes to the theoretical literature on non-monocentric cities (Fujita and Ogawa, 1980; Ogawa and Fujita, 1982; Fujita and Thisse, 2002) and more particularly on that of duocentric cities (Henderson and Mitra, 1996; Fujita, et al., 1997; Smith and Zenou, 1997; Brueckner and Zenou, 2003; Zenou, 2007). Indeed, in the latter, different aspects of duocentric cities have been investigated: formation of subcenters, urban unemployment, spatial mismatch. Here we focus on the impact of differentiated property taxes on urban sprawl and, to the best of our knowledge, this is the first paper that tackles this issue in a nonmonocentric framework..

2 Theory

We now develop our theoretical model in order to examine the connection between the property tax and urban sprawl in a duocentric city.

2.1 The model

City The city is *duocentric*, *closed* and *linear* where the Central Business District (CBD hereafter) is located at the origin (zero) and the Suburban Business District (SBD hereafter) is located at the other end of the city, the city fringe x_f. All land is owned by *absentee landlords*.

The city is depicted in Figure 1. The area $a = c$ (c stands for the city-center) is between $x = 0$ (CBD) to $x = \widetilde{x}$ and only individuals working in the CBD live there. The area $a = s$ (s stands for the suburbs) is between $x = \widetilde{x}$ and $x = x_f$ and only SBD-workers reside there.

[*Insert Figure* 1 *here*]

Firms (land developers) Firms are assumed to consume no space. We consider two types of firms: those located in the CBD or those in the SBD. For the area $a = c, s$, there is a housing industry that has the following production function:

$$Q_a = H(K_a, L_a) = 2\sqrt{K_a L_a} \tag{1}$$

where Q_a is the housing output in area a and L_a and K_a are respectively land and capital (or nonland input) in area a. This function is increasing and concave in each of its arguments and has constant returns to scale, which implies that the production function can be written as:

$$h(S_a) = \frac{Q_a}{L_a} = 2\sqrt{S_a} \tag{2}$$

where $S_a \equiv K_a/L_a$ represents the capital per acre of land or *improvements* per acre in area a and thus $h(S_a)$ is the housing output per acre of land in a. S_a is also referred to as *structural density* (Brueckner, 1987) and is an index of the height of buildings. The function $h(S_a)$ defined by (2) is housing output per acre of land in area a, with $h'(S_a) > 0$ and $h''(S_a) < 0$.

Denote by θ_a the property tax rate in area a. Then, each profit maximizing housing developer solves:[2]

$$\max_{S_a} \left\{ \pi_a = R_H \, 2\sqrt{S_a} - (1+\theta_a)\,(R + rS_a) \right\} \qquad \text{at each } x \in [0, x_f] \tag{3}$$

where π_a is the profit per acre of land in area a, R_H is the rental price per unit of housing service q, R is the rent per unit of land (land cost per acre) and r the price of capital (or the cost per unit of S_a). The city fringe is denoted by x_f and x is the distance to the CBD.

Consumers/Workers There is a continuum of ex ante *identical* workers whose mass is fixed and equal to $\overline{N}$. Among the $\overline{N}$ workers, there are N_c of them working in the CBD, earning an income of y_c, and N_s employed in the SBD, earning y_s, so that

$$\overline{N} = N_c + N_s \tag{4}$$

Each household contains one person. Each individual in area $a = c, s$ chooses z_a and q_a (where z_a and q_a are, respectively, the consumption of the composite good, whose price is taken as the numeraire, and the dwelling size in area a) that maximize his/her utility function under the budget constraint, i.e.

$$\max_{z_c, q_c} U(z_c, q_c) \text{ s.t. } z_c + R_H\, q_c = y_c - t\, x \tag{5}$$

for CBD-workers and

$$\max_{z_s, q_s} U(z_s, q_s) \text{ s.t. } z_s + R_H\, q_s = y_s - t\, (x_f - x) \tag{6}$$

for SBD-workers. Here, t denotes the pecuniary commuting cost per unit of distance. As in Song and Zenou (2006), we assume a quasi-linear utility function, that is:

$$U(z_a, q_a) = z_a + \log q_a \tag{7}$$

In that case, solving (5) and (6) lead to:

$$q_c = q_s = q = \frac{1}{R_H} \tag{8}$$

[2] Observe that it does not matter whether the developer or the urban resident pays the property tax θ_a. The same results would emerge if the residents pay at a rate θ_a, so that the gross-of-tax rent price is written $R_H(1+\theta_a)$. Then, the developer profit will just be $R_H h(S_a) - (R + rS_a)$, with no tax term showing up.

$$z_c = y_c - t\,x - 1 \tag{9}$$

$$z_s = y_s - t\,(x_f - x) - 1 \tag{10}$$

The indirect utility functions can thus be written as:

$$u = y_c - t\,x - 1 - \log R_H \tag{11}$$

$$u = y_s - t\,(x_f - x) - 1 - \log R_H$$

where u is the utility level obtained by all individuals in the city (since all workers are identical and perfectly mobile, they must reach the same utility level in equilibrium), and the bid rent function is given by:

$$R_{H,c}(x, u) = \exp(y_c - t\,x - 1 - u) \tag{12}$$

$$R_{H,s}(x, u) = \exp[y_s - t\,(x_f - x) - 1 - u]$$

Plugging the value of the bid rent in q gives finally

$$q_c(x, u) = \exp(u - y_c + t\,x + 1) \tag{13}$$

$$q_s(x, u) = \exp[u - y_s + t\,(x_f - x) + 1]$$

It is important to observe that, even though the housing consumption q_a is not directly affected by income y_a (see (8)),[3] it is indirectly affected by income through the land rent (see (13)). Indeed, when income increases, the bid rent increases (see (12)) since people are richer. As a result, because housing is more costly, they consume less land and thus reduce their dwelling size. This seemingly counterintuitive result is due to the fact that we analyze the effect of y_a on $q_a(x, u)$ holding u constant.

2.2 The equilibrium

Plugging (12) in (3), the housing developer's program becomes

$$\max_{S_c} \left\{ \pi_c = 2\sqrt{S_c} \exp(y_c - t\,x - 1 - u) - (1 + \theta_c)(R + rS_c) \right\}$$

for firms in area c at each $x \in [0, \widetilde{x}]$ and

$$\max_{S_s} \left\{ \pi_s = 2\sqrt{S_s} \exp[y_s - t\,(x_f - x) - 1 - u] - (1 + \theta_s)(R + rS_s) \right\}$$

for firms in area s at each $x \in [\widetilde{x}, x_f]$. The first order conditions yield:

$$S_c = \frac{\exp[2(y_c - t\,x - 1 - u)]}{(1 + \theta_c)^2 r^2} \text{ for } x \in [0, \widetilde{x}] \tag{14}$$

[3] This is because of the log-linear nature of the utility function, which is defined in (7).

$$S_s = \frac{\exp\left[2\left(y_s - t\ (x_f - x) - 1 - u\right)\right]}{\left(1+\theta_s\right)^2 r^2} \text{ for } x \in [\widetilde{x}, x_f] \tag{15}$$

and thus

$$h(S_c) = 2\frac{\exp\left(y_c - t\,x - 1 - u\right)}{\left(1+\theta_c\right) r} \text{ for } x \in [0, \widetilde{x}] \tag{16}$$

$$h(S_s) = 2\frac{\exp\left[y_s - t\ (x_f - x) - 1 - u\right]}{\left(1+\theta_s\right) r} \text{ for } x \in [\widetilde{x}, x_f] \tag{17}$$

We can now define the *population density* as

$$D_c \equiv \frac{h(S_c)}{q(x, u_c)} = 2\frac{\exp\left[2\left(y_c - t\,x - 1 - u\right)\right]}{\left(1+\theta_c\right) r} \text{ for } x \in [0, \widetilde{x}] \tag{18}$$

$$D_s \equiv \frac{h(S_s)}{q(x, u_s)} = 2\frac{\exp\left[2\left(y_s - t\ (x_f - x) - 1 - u\right)\right]}{\left(1+\theta_s\right) r} \text{ for } x \in [\widetilde{x}, x_f] \tag{19}$$

which is the ratio between square feet of floor space per acre of land and square feet of floor space per dwelling (person). This is a different concept than the *structural density* or *improvements* defined by S_a. As noted above, the improvements (i.e. the intensity of land development) are a measure of building height so a higher S_a means that developers construct higher buildings, containing more housing floor space per acre of land. On the other hand, a higher population density means that either the housing floor space is higher or the dwelling size is lower.

Since $H(.)$ has constant returns to scale, in equilibrium, the housing industry in each area a is such that all firms make zero profit at each x, that is

$$R(x, u, \theta_c) = \frac{\exp\left[2\left(y_c - t\,x - 1 - u\right)\right]}{\left(1+\theta_c\right)^2 r} \tag{20}$$

$$R(x, u, \theta_s) = \frac{\exp\left[2\left(y_s - t\ (x_f - x) - 1 - u\right)\right]}{\left(1+\theta_s\right)^2 r} \tag{21}$$

which implies that

$$\frac{\partial R(x, u, \theta_c)}{\partial x} < 0\ ,\ \frac{\partial^2 R(x, u, \theta_c)}{\partial x^2} \geq 0$$

$$\frac{\partial R(x, u, \theta_s)}{\partial x} > 0\ ,\ \frac{\partial^2 R(x, u, \theta_c)}{\partial x^2} \geq 0$$

This equation gives the bid-rent function for land and is found by solving for R in the zero-profit condition, using (14) and (16) for area c and (15) and (17) for area s.

We can now formally define the equilibrium.

Definition 1 *An urban land-use equilibrium in a duocentric, linear and closed city with absentee landlords is a vector* $(u, \widetilde{x}, x_f, N_c, N_s)$ *such that:*

$$R(\widetilde{x}, u, \theta_c) = R(\widetilde{x}, u, \theta_s) \tag{22}$$

$$R(\widetilde{x}, u, \theta_s) = R_A \tag{23}$$

$$\int_0^{\widetilde{x}} \frac{h(S_c)}{q(x,u)} dx = N_c \tag{24}$$

$$\int_{\widetilde{x}}^{x_f} \frac{h(S_s)}{q(x,u)} dx = N_s \tag{25}$$

$$N_c + N_s = \overline{N} \tag{26}$$

Equation (22) says that the bid rent of CBD- and SBD-workers must intersect at some distance $\widetilde{x}$ while the equation (23) states that the agricultural land rent R_A is equal to the land rent at $\widetilde{x}$. Equations (24) and (25) give the two population constraints. Finally, the last equation is the labor market equilibrium condition. Observe that we focus on a closed city, which implies that the total population $\overline{N}$ is fixed but not N_c and N_s, which are endogenous variables. Observe also that workers living in $\widetilde{x}$ are indifferent between working in the CBD or the SBD. Below $\widetilde{x}$, all residents work in the CBD and $R(x, u, \theta_c) > R(x, u, \theta_s)$ holds for all $x \in [0, \widetilde{x}]$, while beyond $\widetilde{x}$, all residents work in the SBD and $R(x, u, \theta_c) < R(x, u, \theta_s)$ holds for all $x \in [\widetilde{x}, x_f]$. We have the following result.

Proposition 1 *Consider a duocentric, closed and linear city where landlords are absentee. If the utility function is quasi-linear and defined as in (7), the production function* $h(S)$ *is Cobb-Douglas as in (1), then we obtain the following equilibrium values:*

$$x_f^* = \frac{y_c + y_s}{t} - \frac{1}{t} \log \left[\frac{(1+\theta_s)\exp(2y_c) + (1+\theta_c)\exp(2y_s)}{1+\theta_s+1+\theta_c+t\overline{N}/R_A} \right] \tag{27}$$

$$\widetilde{x}^* = \frac{y_c}{t} + \frac{1}{2t} \left[\log \left\{ \frac{(1+\theta_s)}{(1+\theta_c)} \frac{1+\theta_s+1+\theta_c+tN/R_A}{(1+\theta_s)\exp(2y_c) + (1+\theta_c)\exp(2y_s)} \right\} \right] \tag{28}$$

$$u^* = \frac{1}{2} \log \left[\frac{(1+\theta_s)\exp(2y_c) + (1+\theta_c)\exp(2y_s)}{\left(1+\theta_s+1+\theta_c+t\overline{N}/R_A\right)(1+\theta_s)(1+\theta_c)\, rR_A} \right] - 1 \tag{29}$$

$$N_c^* = N - \frac{\left(1+\theta_s+1+\theta_c+t\overline{N}/R_A\right)(1+\theta_s)(1+\theta_c)\, rR_A}{(1+\theta_s)^2\, rt\exp\left[2(y_s+y_c)\right] + (1+\theta_c)\exp(2y_s)} + \frac{(1+\theta_s)\, R_A}{t} \tag{30}$$

$$N_s^* = \frac{\left(1+\theta_s+1+\theta_c+t\overline{N}/R_A\right)(1+\theta_s)(1+\theta_c)\, rR_A}{(1+\theta_s)^2\, rt\exp\left[2(y_s+y_c)\right] + (1+\theta_c)\exp(2y_s)} - \frac{(1+\theta_s)\, R_A}{t} \tag{31}$$

Proof. See the Appendix.

Denote by

$$\phi_\theta \equiv \frac{1+\theta_s}{1+\theta_c}$$

the ratio between the property tax in the suburbs and in the center.

Proposition 2 *We have:*

$$\frac{\partial x_f^*}{\partial \phi_\theta} \gtreqless 0 \Leftrightarrow y_s - y_c \gtreqless \frac{1}{2}\log\left(1+tN\right)$$

$$\frac{\partial x_f^*}{\partial N} > 0$$

$$\frac{\partial x_f^*}{\partial y_c} \gtreqless 0 \Leftrightarrow y_s - y_c \gtreqless \frac{1}{2}\log\phi_\theta$$

$$\frac{\partial x_f^*}{\partial y_s} \gtreqless 0 \Leftrightarrow y_s - y_c \lesseqgtr \frac{1}{2}\log\phi_\theta$$

$$\frac{\partial x_f^*}{\partial t} \gtreqless 0 \Leftrightarrow y_s + y_c \lesseqgtr \frac{tN}{\phi_\theta+1+tN} + \log\left[\frac{\phi_\theta\exp\left(2y_c\right)+\exp\left(2y_s\right)}{\phi_\theta+1+tN}\right]$$

Proof. See the Appendix.

Since the empirical analysis is about the relationship between ϕ_θ and x_f^*, let us comment only this relationship. An increase in the property tax ratio between the suburbs and the city-center, $\phi_\theta \equiv \left(1+\theta_s\right)/\left(1+\theta_c\right)$, does not always decrease the urban sprawl x_f^*. Observe however that if $y_s < y_c$, then $\partial x_f^*/\partial\phi_\theta < 0$, but if $y_s > y_c$, then the sign is indeterminate and depends on the above condition. In order to understand this effect, one has to analyze the effect of a property tax on urban sprawl in a monocentric city (Brueckner and Kim, 2003; Song and Zenou, 2006). Using a similar utility function (a log-linear one), Song and Zenou (2006) showed that this effect was always negative. By remembering our discussion about structural versus population density, the intuition of this result is easy to understand. There are two countervailing effects of an increase of the property tax θ (here θ is the property tax everywhere in the city since there is only one business center) on urban sprawl x_f^*. On the one hand, an increase in θ has a *direct negative* effect on the profit of developers, which accordingly reduces the level of improvements (or structural density). As a result, for a given size of dwellings, buildings are shorter and thus the population density is lower. Because population is fixed (closed city), it has to be that the city increases in size (this is referred to as the *building height effect*). On the other hand, an increase in θ has an *indirect negative* effect on households' housing consumption because the tax on land

and improvements is partly shifted forward to consumers, which yields a higher price of housing and thus a lower dwelling size. Smaller dwellings imply an increase in population density and thus less urban sprawl (this is referred to as the *dwelling size effect*). The net effect is not ambiguous in Song and Zenou, (2006) because consumptions of z (composite good) and q (housing) are highly substitutable since the elasticity of substitution of a log-linear utility function is greater than one. Thus , the dwelling-size effect becomes more important and the net effect is such that an increase in θ decreases urban sprawl.

In the present model with two centers, we still have the same effects but there is a new one. There is now competition in the housing market between CBD- and SBD-workers, which is determined by equation (33) in the Appendix. Let us explain the way this new effect operates. Using (33), one can see that holding the size of the city-center $\widetilde{x}$ constant, an increase in ϕ_θ reduces the city size x_f. Indeed, when θ_s increases, suburban workers reduce their bid rent (see (21)) while CBD-workers are not directly affected and thus do not directly modify their bid rent (see (20)). However, because the competition for housing between CBD- and SBD-workers is now less fierce, CBD-workers decrease their bid rent. Since $\widetilde{x}$ is held constant and the two bid rents has to intersect at $\widetilde{x}$, the size of the city x_f has to decrease. This effect is depicted in Figure 2.

$[Insert\ Figure\ 2\ here]$

The same intuition applies for a decrease θ_c, which decreases first the bid rent of CBD-workers and then of SBD-workers. We can solve the general problem where both $\widetilde{x}$ and x_f are endogenous by using the population constraints in the city-center (equation (24)) and in the suburbs (equation (25)). In that case, it is not always true that when the ratio ϕ_θ increases, the city-center $\widetilde{x}$ and the city x_f decrease because of general equilibrium effects. Now if we add the effects described above (i.e. the building height effect and the dwelling size effect) to this one (i.e. the housing competition effect), then the net effect is ambiguous. Proposition 2 gives a condition under which this effect is not ambiguous.

3 Developing a national sample of effective tax rates

We would like now to test the main result of our theoretical model, i.e. the impact of ϕ_θ, the ratio between the property tax in the suburbs and in the center, on the spatial extent of urbanized areas. We begin our analysis by presenting the steps involved in developing our sample of effective tax rates.

3.1 Data sources

For each urbanized area[4] in the U.S., we further divide the area between central city and suburb. Generally in an urbanized area, there are various taxing entities such as county, township, city, town, school, and special taxing districts. We thus need to construct the aggregated effective tax rates for the central city and the suburb. We use "central place"[5] as a proxy for central city. The rest of area in an urbanized area is then defined as "suburb" in the study.

To construct the aggregated effective tax rates for the central city and the suburbs, we first collect effective tax rates imposed at different levels of taxing jurisdictions in an urbanized area — counties, cities, townships, and school districts. We do not collect effective tax rates from special districts such as fire, water, sewer, etc. as those tax rates are generally not reported by the state agencies. Since special districts are formed to provide services to the inhabitants of a limited area, we argue that the omission of the tax rates from special districts would not have a significant impact on the results of this study.

Data on the effective tax rates from counties, cities, townships and schools can be collected either from states or local government units. Many state level units, such as the Department of Taxation and Association of County Commissioners, conduct tax rate surveys to collect effective tax rates from various localities and have made effective tax rates available on their websites.[6] As one of the main purposes of collecting tax rates by the state is to offer a common standard for the comparison of tax rates among taxing jurisdictions, these rates are thus comparable across areas and states. Generally, the effective tax rates are obtained by adjusting the nominal tax rate with the sales/assessment ra-

[4]According to US Census Bureau, urbanized areas are defined as cities with 50,000 or more inhabitants and their surrounding densely settled urban fringe, incorporated or unincorporated.

[5]According to US Census Bureau, a central place is defined as the core incorporated place(s) or a census designated place of an urban area, usually consisting of the most populous place(s) in the urban area plus additional places that qualify under Census Bureau criteria. If the central place is also defined as an extended place, only the portion of the central place contained within the urban area is recognized as the central place.

[6]Examples of these websites include:

North Carolina: http://www.ncacc.org/taxrate.htm

Illinois: http://www.revenue.state.il.us/Publications/LocalGovernment/00PTAX50.pdf

New York: http://urban.nyu.edu/research/etr/etr-nyc-1999.pdf tio, which is estimated and determined by the state agencies. For those states without available information online, we directly contact the local government units to obtain data on the effective rates imposed by the local jurisdictions such as the counties, cities and school districts.

Finally, in order to construct the aggregated effective tax rate for both the central city and the suburbs in an urbanized area, we also collect spatial datasets which contain the boundaries of central places and urbanized areas and of various taxing jurisdictions such as counties, cities, townships, and school districts.[7]

3.2 Geographic Information System (GIS) methods

To distill a single value for the central city and for the suburbs from the tax rates imposed by various taxing entities respectively, we then create a weighted average of tax rates by coalescing input tax rates from various jurisdictions based on the localities' spatial relationships within the central city and the suburb. Next, we describe the steps involved in constructing the aggregated tax rate for the central city and the suburbs in each urbanized area. As an example of our approach, Figure 3 presents three levels of tax rates levied in the central city of Salem, OR Urbanized Area: county, city, and school district.

[*Insert Figure* 3 *here*]

First, we use GIS techniques to intersect the boundaries of different taxing jurisdictions with the boundary of the central city of Salem, OR Urbanized Area and obtain the proportion of the central city within any given county, city, or school district. Second, we calculate the property tax rates by each of the three taxing jurisdictions: county, city, and school district.

Specifically, we show that the central city of Salem, OR Urbanized Area falls into two counties: Marion and Polk – with 90% of the central city in Marion County and the rest in Polk County. These two counties impose different tax rates and tax assessment ratios. To obtain the effective tax rate for the central city at the county level, we sum the effective tax rates (which are the product of tax rates and ratios) from the two counties adjusted by their area proportions.[8]

[7] These data are available from the U.S. Census, or can be purchased from GeoCommunity (a GIS data depot).

[8] The aggregate tax rate at the county level is obtained by: Marion County Tax rate $\times$ Marion County Tax assessment ratio $\times$ Proportion of urbanized area in Marion County + Polk County Tax rate $\times$ Polk County Tax assessment ratio $\times$ Proportion of urbanized area in Polk County $= 5.487 \times 0.805 \times 90\% + 3.663 \times 0.796 * 10\% = 4.26$.

We also show that the central city of Salem, OR Urbanized Area contains three cities, Salem, Keizer, and Turner, and that 85% of the central city is in Salem, 9% is in Keizer and the rest is located in Turner. To calculate the effective tax rate at the city level, we also need to find out which county the city is located in since we also need to apply the county tax assessment ratio in the calculation. For example, the city of Salem is in both Marion and Polk counties while the cities of Keizer and Turner are only in Marion County. Thus for the city of Salem, GIS techniques are employed to obtain the proportion of the central city that is in the city of Salem, but in different counties. We show that for the 85% of the central city that is in Salem, 78% is in Marion County and 7% is in Polk County. To obtain the effective tax rate for the central city at the city level, we sum the effective tax rates (which are the product of city tax rates and county tax assessment ratios) from the three cities, adjusted by their area proportions.[9] The strategy of computing the effective tax rate at the city level applies to the calculation of the effective tax rate at the school district level. The calculations indicate that the effective tax millage rates levied by the county, city, and school district are 4.26, 5.20, and 5.31, respectively.[10] Finally, we sum up these three effective tax rates at different levels to obtain the aggregated effective tax millage rate for the central city of Salem, OR Urbanized Area, which is 14.87.

We repeat the steps for the construction of effective property tax rate for the suburbs in Salem urbanized area. Using this approach, we constructed the effective tax rates for the central cities and the suburbs in 448 urbanized areas in the U.S.[11]

[9] The aggregated tax rate at the city level is obtained by: [(Salem Tax rate × Marion County Tax assessment ratio × Proportion of central city in Salem and in Marion County + Salem Tax rate × Polk County Tax assessment ratio × Proportion of central city in Salem and in Polk County) + (Keizer Tax rate × Marion County Tax assessment ratio × Proportion of central city in Keizer) + (Turner Tax rate × Marion County Tax assessment ratio × Proportion of central city in Turner)] $= [(6.852 \times 0.805 \times 78\% + 6.852 \times 0.796 * 7\%) + (3.629 * 0.805 \times 9\%) + (5.311 \times 0.805 * 6\%)] = 5.20$.

[10] A millage is a unit equal to one thousandth. Thus, a tax millage rate of 14.57 equals to 1.457%.

[11] We excluded those urbanized areas with a population size larger than five million as they contain too many localities which complicate the calculations.

4 Data and empirical test

4.1 Variables and data

An empirical test based on the above theoretical analysis is extremely useful to facilitate the debate on the relationship between differentiated property tax and urban development. In particular, we compute the ratio of property tax rate in suburb to the rate in central city. A direct test of Propositions 1 and 2 is to test equation (27), which econometric counterpart can be written as:[12]

$$x_{f,i} = \alpha_0 + \alpha_1\,\phi_{\theta,i} + \alpha_2\,N_i + \alpha_3\,R_{A,i} + \alpha_4\,y_i + \alpha_5\,T_i + \varepsilon_i \tag{32}$$

where i is an index for the urban area, T_i is the total commuting cost in i, and ε_i is a white noise error term. Contrary to the model, we do not split the income and the population into two areas of the urban area (i.e., the central city and the suburbs). Rather, we use data on the total income and the total population in the urban area.[13]

Mills (1998) also provides a justification for including the explanatory variables N_i, $R_{A,i}$, T_i to determine the urban spatial extent. The intuition of including these variables is also stated in Brueckner and Fansler (1983)'s study. A recent study by McGrath (2005) confirms the validity of this set of variables. An increase in the urban population would increase the urban spatial extent since more people would require more housing. An increase in agricultural land rent would lead to a higher opportunity cost of urban land and thus, make the city more compact. A higher level of income would imply an increase in housing demand and thereby, leads to a larger city. Finally, an increase in commuting cost would lower disposal income at all locations and thus, reduce city size. Given the confluence of an expanding population, rising incomes, and falling commuting costs, it is not surprising that most U.S. cities have expanded rapidly in recent decades.

We then perform a regression analysis to examine the effect of differentiated property tax rates on the spatial extent of urbanized areas. This analysis allows us to isolate the effects of property tax on urban size while controlling for other factors.

There is one potential problem in assuming the exogeneity of the property tax: from our theoretical model, we see that the property tax leads to two countervailing effects, which may lead to a reduction in the size of cities. On the other hand, urban sprawl will have an effect on the property tax. For example,

[12] The econometric equation (32) is in fact a linear approximation of (27).

[13] We have also tested the model with variables on income and population disaggregated by the central city and the suburbs. However, the disaggregated variables are not significant.

as demand for supporting infrastructure such as sewer-lines, waterlines, gas lines, phone lines, streets, and gutters increases, some jurisdictions located far away from existing centers of infrastructure, may raise the property tax rates to generate revenues in an attempt to cover service costs (Song and Zenou, 2006). However, observe that because we use the ratio $\phi_{\theta,i}$ and not the level of the property tax,[14] we do not need to instrument this variable because it enters in a non-linear way in equation (32).

We describe the data used to construct the variables here. In the regression model, the dependent variable is the size of urbanized area and is measured by the size, in acres, of the urbanized area in the year 2000. The population variable represents the 2000 urbanized area population. The income variable is a measure of the 2000 median household income. To construct the commuting cost variable, government expenditure on transportation per person driving to work in 1997 is used as a proxy. Other things being equal, a higher value of government expenditure on transportation would be associated with ease of transportation system usage and a lower level of commuting costs. As data on government expenditure on transportation is available at county level, we construct a weighted average of government expenditure on transportation for each urbanized area based on the area proportions of counties in relation to the urbanized area using GIS techniques. Similarly as data on agricultural land rent is only available at county level, we construct a weighted average of median agricultural land value per acre for each urbanized area. Finally, as mentioned above, $\phi_{\theta,i}$, the ratio of 1997 property tax rate in suburb to the rate in central city is constructed according to the steps described in Section 3. Note that we lag the ratio by three years because the effect of differentiated property tax rates on the size of an urbanized area is not instantaneous, but rather takes time.

Data sources and measurements of the dependent and independent variables are summarized in Table 1. Summary statistics of these variables are presented in Table 2.

[*Insert Tables* 1 *and* 2 *here*]

4.2 Empirical results

Given that the theory provides no guidance as to the functional form of the estimating equation, the empirical work makes use of the Box–Cox transfor-

[14] Song and Zenou (2006) use the level of the property tax in the urbanized area and thus intrument this variable with the magnitude of state aid to schools, since this variable is correlated with the property tax rate but not with the urban size.

mation. The optimal value of the functional form parameter λ equals 0.46, indicating that a square-root transformation of the variables is appropriate.

Regression results using OLS are presented in Table 3. Results indicate that the signs of most estimated coefficients conform to the expectations from the theory. In particular, the influence of the ratio of property tax rate in suburbs to the rate in central city on the spatial extent of urban areas is of primary interest to this research. The coefficient of the property tax ratio is negative and statistically significant. The result supports what has been predicted by the theory (Propositions 1 and 2) but giving a precise sign to the relationship between ϕ_θ and x_f^*: the lower the property tax rate in suburbs in comparison to the rate in central city, the larger the spatial scale of urbanized areas.

Concerning the other explanatory variables, the population and income variables have positive and significant coefficients, indicating that the spatial size of urbanized areas is an increasing function of the population and income. The result also shows that the expenditure on transportation variable has a positive and significant coefficient. Here, the government expenditure in transportation is used as a proxy for commuting costs. Thus, since a higher spending level in transportation is designed to improve local transportation and thus to lower commuting costs, the result indicates that there is a negative relationship between the urban size and the commuting costs. The estimated coefficient of agricultural rent variable is not significant. The poor performance of the agricultural rent variable may be a sign that the constructed weighted average of agricultural land rent for the urbanized area is less reflective of the actual agricultural land rent at the periphery of the urbanized area.

[*Insert Table* 3 *here*]

5 Conclusion

This paper has examined the relationship between the differentiated property tax and urban sprawl through both theoretical and empirical analyses. The theoretical model predicts a negative relationship between these two variables. Based on a dataset of the effective property tax rates, using GIS method for central cities and suburbs in 448 urbanized areas, the empirical analysis estimates a regression equation relating an urbanized area's size to a ratio of property tax rate in the suburbs to the rate in central city and other control variables, such as population, income, agricultural rent, and transportation expenditure. Results from the empirical analysis are consistent with findings from the theoretical reasoning, suggesting that lower tax rate in suburbs can induce more developments and thus urban sprawl.

The finding has important policy implications for the US urban development. As varying property tax rates within US urban areas is a common feature, it is essential to note the associated effects of fragmented urban areas on urban development patterns. This study has demonstrated that varying property tax rates could induce spillover of development outwards, given the level of fragmentation within US urban areas. For this reason, remedies for urban sprawl relying on property tax reforms could be more effective if executed at the urban scale.

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Appendix

Proof of Proposition 1. Solving equation (22) using (20) and (21) yields:

$$\widetilde{x} = \frac{x_f}{2} + \frac{1}{2t}\left[y_c - y_s + \log\left(\frac{1+\theta_s}{1+\theta_c}\right)\right] \tag{33}$$

Solving equation (23) using (21) gives:

$$u = y_s - 1 + t\,(\widetilde{x} - x_f) - \log(1+\theta_s) - \frac{1}{2}\log(rR_A) \tag{34}$$

Furthermore, solving equation (24) using (18) leads to:

$$N_c = \frac{\exp[2(y_c - 1 - u)]}{(1+\theta_c)\,rt}\left(1 - \exp[-2t\,\widetilde{x}]\right) \tag{35}$$

while solving (25) using (19) gives:

$$N_c = N - \frac{\exp[2(y_s - 1 - u)]}{(1+\theta_s)\,rt}\left(1 - \exp[2t\,(\widetilde{x} - x_f)]\right) \tag{36}$$

Now, by combining (34) and (36), we obtain the following relationship between N_c and u:

$$N_c = N - \frac{1}{(1+\theta_s)\,rt\exp[2y_s]\exp[2(1+u)]} + \frac{(1+\theta_s)\,R_A}{t} \tag{37}$$

which means that

$$N_s = \frac{1}{(1+\theta_s)\,rt\exp[2y_s]\exp[2(1+u)]} - \frac{(1+\theta_s)\,R_A}{t}$$

By combining (33) and (34), we obtain the following relationship between $\widetilde{x}$ and u:

$$t\widetilde{x} = y_c - \log(1+\theta_c) - \frac{1}{2}\log(rR_A) - (1+u) \tag{38}$$

By plugging (33) into (38), we obtain the following relationship between x_f and u:

$$tx_f = y_c + y_s - \log[(1+\theta_s)(1+\theta_c)] - \log(rR_A) - 2(1+u) \tag{39}$$

which can be written as

$$\exp[2(1+u)] = \frac{\exp[y_c + y_s - tx_f]}{(1+\theta_s)(1+\theta_c)(rR_A)} \tag{40}$$

By plugging (33) into (35), we obtain:

$$N_c = \frac{\exp[2(y_c - 1 - u)]}{(1+\theta_c)\,rt} - \frac{\exp[y_s + y_c - 2(1+u) - tx_f]}{(1+\theta_s)\,rt} \tag{41}$$

By plugging (33) into (36), we obtain:

$$N_c = N - \frac{\exp\left[2\left(y_s - 1 - u\right)\right]}{\left(1+\theta_s\right)rt} + \frac{\exp\left[y_c + y_s - 2\left(1+u\right) - tx_f\right]}{\left(1+\theta_c\right)rt} \tag{42}$$

Now, by combining (41) and (42), we obtain the following relationship between u and x_f:

$$\begin{aligned} & \exp\left[2\left(1+u\right)\right] \qquad (43) \\ = \; & \frac{\left(1+\theta_s\right)\exp\left[2y_c\right] + \left(1+\theta_c\right)\exp\left[2y_s\right] - \left(2+\theta_s+\theta_c\right)\exp\left[y_c+y_s-tx_f\right]}{\left(1+\theta_c\right)\left(1+\theta_s\right)rtN} \end{aligned}$$

Finally, by combining (40) and (43), we obtain the equilibrium value of the city-fringe:

$$x_f^* = \frac{y_c + y_s}{t} - \frac{1}{t}\log\left[\frac{\left(1+\theta_s\right)\exp\left(2y_c\right) + \left(1+\theta_c\right)\exp\left(2y_s\right)}{1+\theta_s+1+\theta_c+tN/R_A}\right]$$

We can now calculate the other equilibrium values. By plugging x_f^* in (39), we have:

$$u^* = \frac{1}{2}\log\left[\frac{\left(1+\theta_s\right)\exp\left(2y_c\right) + \left(1+\theta_c\right)\exp\left(2y_s\right)}{\left(1+\theta_s+1+\theta_c+tN/R_A\right)\left(1+\theta_s\right)\left(1+\theta_c\right)rR_A}\right] - 1$$

By plugging x_f^* in (33), we obtain:

$$\widetilde{x}^* = \frac{y_c}{t} + \frac{1}{2t}\left[\log\left\{\frac{\left(1+\theta_s\right)}{\left(1+\theta_c\right)}\frac{1+\theta_s+1+\theta_c+tN/R_A}{\left(1+\theta_s\right)\exp\left(2y_c\right) + \left(1+\theta_c\right)\exp\left(2y_s\right)}\right\}\right]$$

Finally, plugging u^* into (37) gives:

$$N_c^* = N - \frac{\left(1+\theta_s+1+\theta_c+tN/R_A\right)\left(1+\theta_s\right)\left(1+\theta_c\right)rR_A}{\left(1+\theta_s\right)^2 rt\exp\left[2\left(y_s+y_c\right)\right] + \left(1+\theta_c\right)\exp\left(2y_s\right)} + \frac{\left(1+\theta_s\right)R_A}{t}$$

and since $N_s^* = \overline{N} - N_c^*$, we have:

$$N_s^* = \frac{\left(1+\theta_s+1+\theta_c+tN/R_A\right)\left(1+\theta_s\right)\left(1+\theta_c\right)rR_A}{\left(1+\theta_s\right)^2 rt\exp\left[2\left(y_s+y_c\right)\right] + \left(1+\theta_c\right)\exp\left(2y_s\right)} - \frac{\left(1+\theta_s\right)R_A}{t}$$

∎

Proof of Proposition 2. First, in order to express x_f^* in terms of the ratio $\phi_\theta \equiv (1+\theta_s)/(1+\theta_c)$, we normalize the agricultural land rent as follows: $R_A \equiv 1/(1+\theta_c)$. It is easy to verify that this normalization does not affect qualitatively the comparative statics results below. As a result, equation (27) is equivalent to:

$$x_f^* = \frac{y_c + y_s}{t} - \frac{1}{t}\log\left[\frac{\phi_\theta \exp(2y_c) + \exp(2y_s)}{\phi_\theta + 1 + tN}\right] \tag{44}$$

Similarly, in order to express $\widetilde{x}^*$ in terms of the ratio $\phi_\theta \equiv (1+\theta_s)/(1+\theta_c)$, we use the same normalization: $R_A = 1/(1+\theta_c)$. Thus, equation (28) can be written as:

$$\widetilde{x}^* = \frac{y_c}{t} + \frac{1}{2t}\left[\log\left\{\frac{\phi_\theta \exp(2y_c) + \exp(2y_s)}{\phi_\theta + 1 + tN}\right\}\right] \tag{45}$$

By totally differentiating (44), it is straightforward to show that:

$$\frac{\partial x_f^*}{\partial \phi_\theta} \gtreqless 0 \Leftrightarrow y_s - y_c \gtreqless \frac{1}{2}\log(1+tN)$$

$$\frac{\partial x_f^*}{\partial N} > 0$$

$$\frac{\partial x_f^*}{\partial y_c} \gtreqless 0 \Leftrightarrow y_s - y_c \gtreqless \frac{1}{2}\log\phi_\theta$$

$$\frac{\partial x_f^*}{\partial y_s} \gtreqless 0 \Leftrightarrow y_s - y_c \lesseqgtr \frac{1}{2}\log\phi_\theta$$

$$\frac{\partial x_f^*}{\partial t} \gtreqless 0 \Leftrightarrow y_s + y_c \lesseqgtr \frac{tN}{\phi_\theta + 1 + tN} + \log\left[\frac{\phi_\theta \exp(2y_c) + \exp(2y_s)}{\phi_\theta + 1 + tN}\right]$$

Also, by totally differentiating (45), it can be shown that:

$$\frac{\partial \widetilde{x}^*}{\partial \phi_\theta} \gtreqless 0 \Leftrightarrow \exp[2(y_s - y_c)] \lesseqgtr 1 + tN$$

∎

Figure 1. Land-use equilibrium in a duocentric city

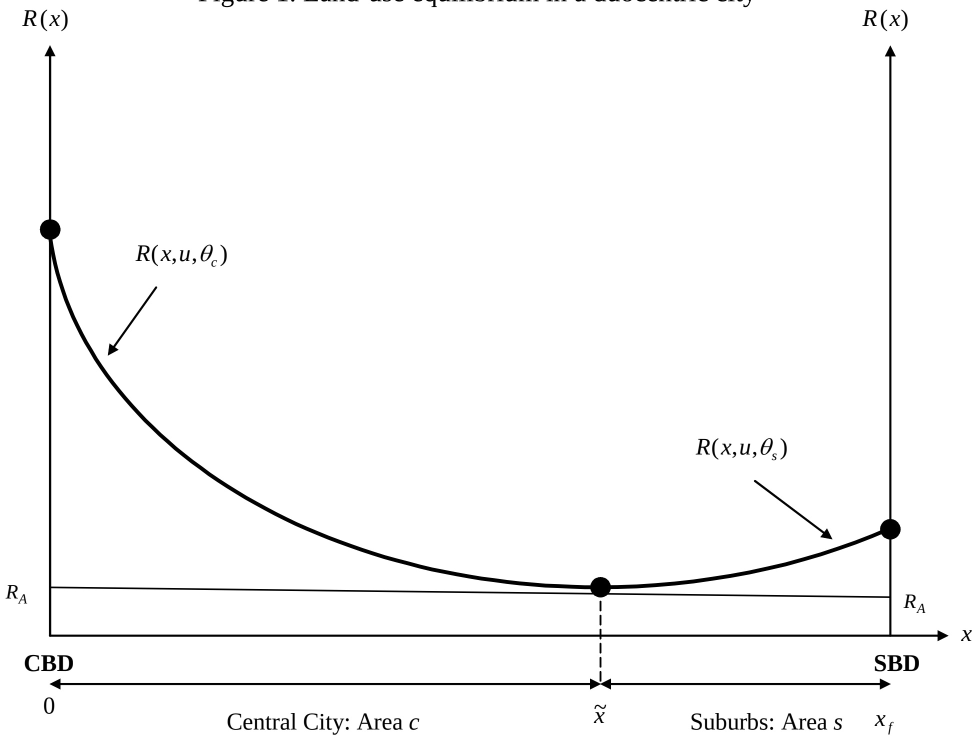

Figure 2. Effect of an increase in θ_s holding $\tilde{x}$ constant

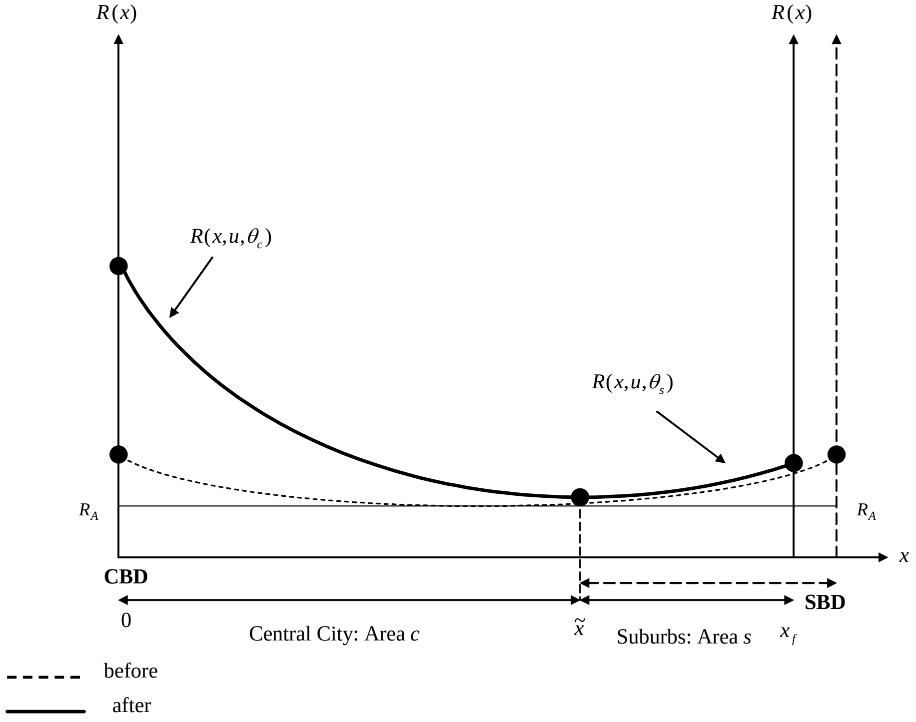

Figure 3. Levels of taxation in the Central Place of Salem Urbanized Area, OR

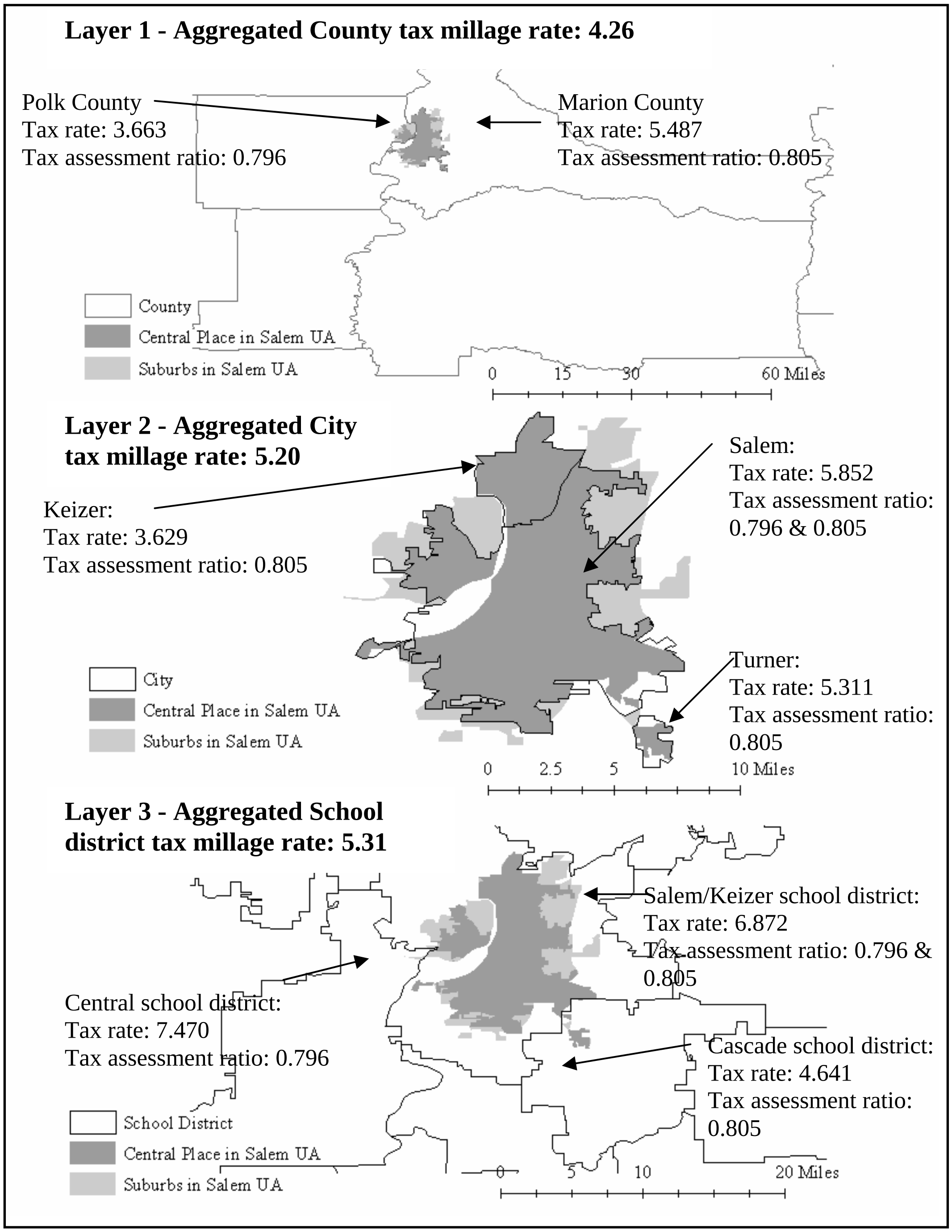

Table 1. Dependent and Independent Variables and Measurements

Variables (Variable Name)	Measurements (Data Source)
Dependent Variable	
Size of urbanized area (UA)	The spatial extent of land area in the urbanized area in acres in 2000 (U.S. Census).
Independent Variables	
Population (POP)	2000 Urbanized area population (U.S. Census).
Income (INCOME)	2000 Median household income.
Agricultural land rent (AGVAL)	1997 Median agricultural land value per acre for the county containing the urbanized area (U.S. Census of Agriculture/National Agricultural Statistics Service and GIS operation).
Government expenditure on transportation (TRANS)	1997 Transportation expenditure per person who drives to work (U.S. Census of Governments and GIS operation).
Ratio of Property Tax in suburb to the rate in central city (TAXRATIO)	Ratio of weighted average property tax millage rates for suburb and central city in 1997 (U.S. Census, Web survey, Secondary Data sources and GIS operation).

Table 2. Descriptive Statistics of the Variables

Variable (Unit)	Minimum	Maximum	Mean	Std. Deviation
UA (acres)	7,742	1,256,051	90,112	141,797
POP	49,776	4,918,839	333,239	635,474
INCOME (dollars)	11,381	79,614	39,882	10,192
AGVALUE (dollars)	0	224,006	1,418	10,954
TRANS (dollars)	4	1,481	263	254
TAXRATIO (ratio)	0.45	1.54	0.84	0.15

Sample size: 448

Table 3. Regression Results

	Coefficients	t-statistics
constant	131.1045 ***	5.58
	(21.34)	
POP	0.00020***	20.31
	(0.00)	
INCOME	0.00170***	4.73
	(0.00)	
AGVAL	0.00004	0.41
	(0.00)	
TRANS	0.1413***	9.15
	(0.01)	
TAXRT	-35.8861***	4.56
	(18.36)	
R SQUARE	0.85	

Notes:
Standard errors are in parentheses.
* Significant at 10% level
** Significant at 5% level
*** Significant at 1% level