Parcel-specific land valuation at the metropolitan scale: An option-theoretic approach

Presentation before the Lincoln Institute David C. Lincoln Fellowship Symposium:
Measuring the Value of Land

January 29, 2021

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Our objective:

- To develop, calibrate, and test a methodology to estimate the presence and magnitude of *one component of land value* that in some cases may be significant in absolute value and/or as a percentage of total land value (especially in urbanizing areas or older areas undergoing market dynamics)
- To be able to be applied to *a single site* at *a specific point in time*
- To provide estimates that *can be validated* using supplemental data and analysis
- In addition to providing an estimate of value, to *provide additional guidance* for developers, investors, assessors decision making: e.g. timing of activity, density and nature of development, redevelopment, etc.

Our methodology:

- Application of *real option theory* to the case of real estate investment decision making and its effect on land values
- Investment decisions over time that can affect the *"option value"* of land:
  - Purchase of vacant land
  - Initial development
  - CAPEX investments
  - Renovation
  - Redevelopment/adaptive reuse
  - Abandonment
- In each case developer/investor/owner has the *right (but not obligation)* to undertake the specified activity. Includes timing and density/nature of activity as well as price
- Option value derived from the *volatility* associated with future property value and construction cost outcomes. *Higher* volatility implies *higher* option value (*not lower*) because it represents value of embedded option.

More facts about option valuation:

- Options can get *complex*, e.g. compound, sequential (e.g., development and future redevelopment). Thus they can interact with each other in valuation, optimal timing and intensity.
- Option valuation is *applied broadly* in financial markets, less so in real estate. One issue is *replication* requirement and assumptions of *perfect and complete capital markets*. Much analysis and recent innovations have reduced these problems
- Option valuation can and does have the ability to provide *sufficiently accurate estimates* of option value so long as model is correctly specified, parameters are correctly estimated and validation is undertaken. It is, however, primarily a *normative*, rather than a *descriptive* approach (like the DCF income approach vs. the sales comparison approach)
- There is now a substantial legacy of research applying option pricing techniques to real estate, both theoretical and empirical, that allows us to build upon the best of this to achieve our objectives when constructing and testing our model

What unique contributions do we intend to make?

- Intent to respond directly to Lincoln Institute Request: To develop a methodology for the valuation of land that is able to be
  - *Understood* by valuation professionals,
  - *Transparent* and not a “black box”
  - To be applied to *individual parcels* at the *local submarket level* without *excessive or unavailable required data* inputs,
  - To be applied to *all parcels*, regardless of whether they are vacant or built upon or their size or use or their stage their life cycle (excluding public lands or any parcels restricted from the private market)
  - To be *accurate* within the normal bounds for assessment or appraisal purposes
  - To be *consistent* with economic and appraisal theory.
- Our model is intended to accomplish all of the above and in addition the following:
  - Provide information on anticipated *time to develop or redevelop* and on *type and intensity* of such development
  - Incorporate *finer institutional detail* on the nature of costs incurred upon development/redevelopment
  - Recognize the important role that *covariation among inputs* (e.g., property values and construction costs) can have on value estimates
  - Possess *greater flexibility* to handle the anticipated succession of more than one event (e.g., development followed by later redevelopment)

HOWEVER…Certain facts must also be recognized

- The development option value of land is *only one component* of land value; there is also intrinsic value, unrelated to volatility
  - It is expected to be small in absolute value on the distant urban fringe and at a time near and just after development and redevelopment events, but never zero so long as compound options to be exercised in the future are present. KEEP IN MIND THAT IT FIRST APPEARED AT THE TIME OF THE FOUNDING OF FORT MCDOWELL IN 1865 AND THE AGRICULTURAL SETTLEMENT BY SWILLING IN 1868!
  - Though often small in an absolute sense, it can still represent a high proportion of total land value on the urban fringe, reaching its maximum at a point in time in which the NPV for initial development becomes positive, then declining to the point of optimal development at which time the development option reaches zero (the "smooth pasting" condition)
- Its magnitude can be *highly sensitive* to certain inputs, requiring a higher degree of care in their estimation
- In its current form it typically assumes the stochastic processes driving property values and construction costs are *static* over time and state space and result in *log normal* return distributions, which may not always reflect reality

Introducing our model: The Cox-Ross-Rubenstein (CRR) binomial option pricing model applied to land

- Forerunner was Titman (1985). CRR itself first applied to real estate development and land by Geltner (1989). More recent efforts by many others.
- Other option-theoretic models possible: Black-Scholes (BS), Partial Differential Equation (PDE), and Samuelson-McKean (SM). All found to converge to same results in the limit.
- Solutions for BS, PDE, and SM are primarily numerical, except in simplest cases for single option (development). Childs, Riddiough, and Triantes (1998) and Williams (1991) use PDE methods to solve for redevelopment option analytically
- CRR models can be solved analytically using dynamic programming algorithm, even with compound options. (Though even these may be more efficiently solved using Monte-Carlo simulation or other numerical methods.)
- First empirical efforts to identify magnitude of land option value by Quigg (1993); more recently others. These primarily use PDEs to model option, then modified hedonic models to estimate land option value.
- We have settled upon an analytic solution using the CRR model for multiple options (initially development and subsequent redevelopment). CRR has been identified in the finance literature as more flexible for complex/compound options, so is more prevalent in practice.
- Full disclosure: Although we have adopted an analytic solution for compound options, it may well be more efficient to use Monte Carlo or other numerical approaches for multiple compound options beyond two. We restrict our analysis only to the development and subsequent redevelopment options

The basic CRR binomial model illustrated - Step 1: Rolling forward fundamental asset (property value)

r_f = Risk-free rate

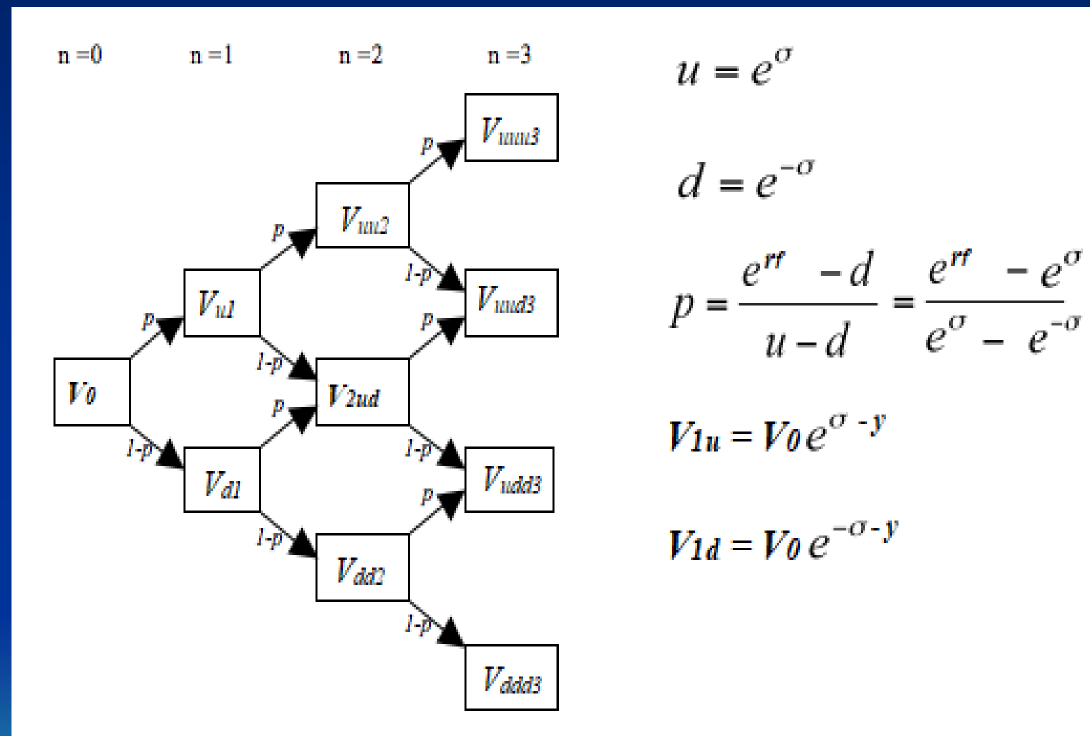
y = NOI (dividend) rate

σ_V = Volatility of property value *ex dividend*

u, d = Magnitude of up, down movement

p, 1-p =Synthetic probability of up, down movement

V_{ij} = Property value in period *j* and state *i*



The basic CRR binomial model illustrated - Step 2: Creating derivative asset (land value) and folding back for land valuation with development option

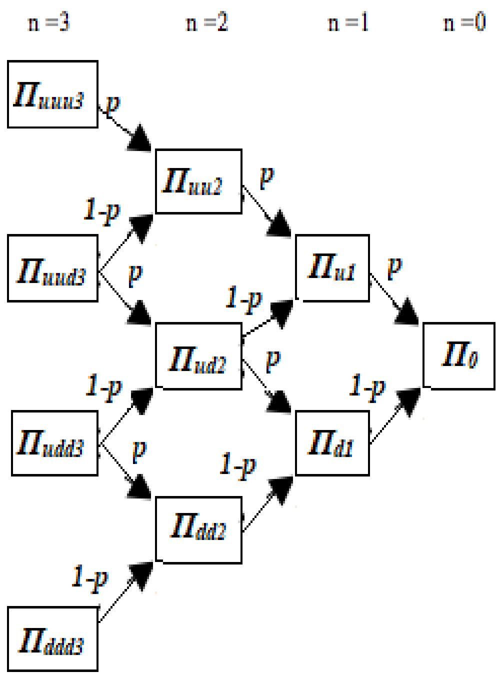
V_{ij} = Property value in period *j* and state *i*

K_{ij} = Construction cost in period *j* and state *i*

Π_{ij} = Option land value in period *j* and state *i*

Π_0^* = Equilibrium Option land value in period 0

η^* =Equilibrium land value ratio in period 0



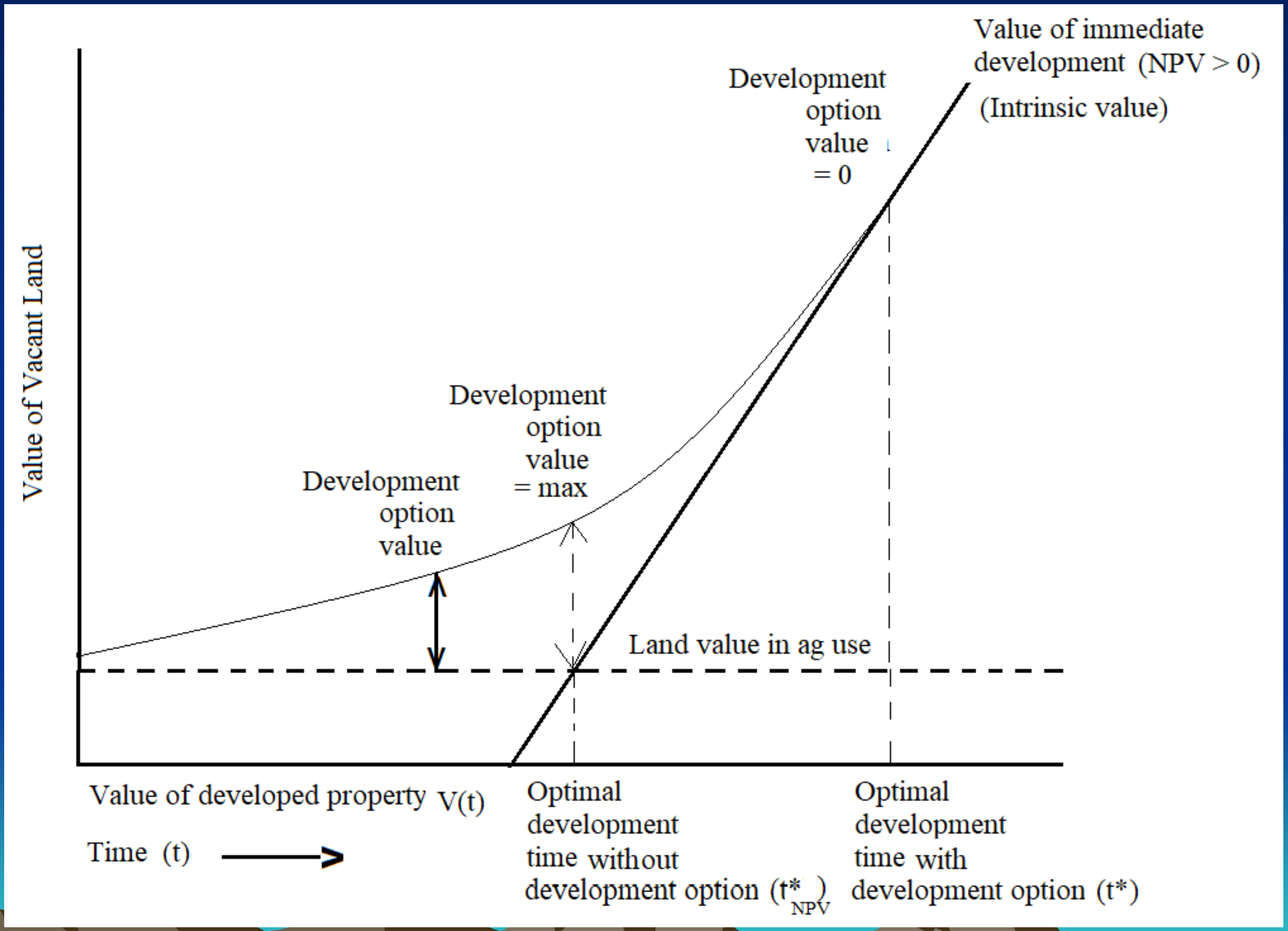
$$\Pi_{iN} = Max[(V_{iN} - K_{iN}), 0]$$

$$\Pi_{ij} = Max\{(V_{ij} - K_{ij}), [p(V_{i,j+1} - K_{i,j+1}) + (1 - p)(V_{i+1,j+1} - K_{i+1,j+1})]e^{-r}, 0\}$$

$$If\ (V_{ij} - K_{ij}) > [p\ (V_{ij+1} - K) + (1 - p)\ (V_{i+1,j+1} - K)]e^{-r} \Rightarrow DEVELOP$$

$$\Pi_0 \Leftrightarrow (V_0 - K_0) \Rightarrow \Pi_0 = \Pi_0^* \text{ and } \Pi_0 / V_0 = \eta^*$$

The basic CRR binomial model illustrated: Components of land value (development option value and intrinsic value) over time



The basic CRR binomial model: Extension 1
Stochastic construction costs

- Past CRR applications assumed *K* was fixed in state space or both time and state space. Avoided problem of correlation between *V* and *K*.
- The joint probability matrix *h(V, K)* is under-identified. Cannot be solved for uniquely.
- Solution: Made use of the Margrabe/Fisher transformation of the stochastic fundamental asset from *V to V/K* (the *price per-unit construction cost*)
- Derivative security *V- K* becomes *V/ K -1*. In this form the appropriate parameter for dispersion over time for *V/K* ($\sigma_{V/K}$) converges to

$$\sigma_{V/K}^2 = \sigma_V^2 + \sigma_K^2 - 2\,\rho(V, K)\,\sigma_V\,\sigma_K$$

- The problem then becomes the empirical estimation of σ_V, σ_K, and $\rho(V, K)$.

The basic CRR binomial model: Extension 2
Adjustments to existing improved property price - Depreciation

- Depreciation of improved property *V* not relevant for *new development,* as price index over time assumes a newly developed structure at each point of time and state space. However, highly relevant, in fact necessary or even fundamental, for *existing development* to explain and price the redevelopment option.

- Modeled by a constant proportional value decrement δ (geometric functional form) $V_{ij+1} = V_{ij}\, e^{-\delta}$, consistent with empirical findings by Bokhari and Geltner (2015) and others.

- Note: depreciation affects the *whole* property, not simply the improvements. Not confined to *physical* and *functional* depreciation relating to the *improvements*, but also may be due to the impact of its *surroundings* (i.e., *environmental* depreciation), which could also affect land values.

- Caveat: δ *a*ssumed constant over all time and state space. Realistic from generalized *ex ante* perspective, but may not reflect reality in certain situations. Possible dynamic extensions to estimation?

The basic CRR binomial model: Extension 3
Adjustments to existing improved property price – Dividends

- Our basic CRR binomial land option pricing model above did not include the "leakage" of cash dividends (net income or NOI) payable periodically to investors.

- These are quite commonly incorporated in CRR models applied to option valuation in finance (when they represent actual payouts), and rightly should be applied to such models applied to the development (and redevelopment) land development option for real estate.

- Most of such models applied to real estate development (e.g. Geltner) do include dividends in the form of deriving an *ex dividend* value for the improved property. (These in fact are necessary to later derive estimates of optimal time to develop.)

- Modeled by a constant proportional value decrement *y*: $V_{ij+1} = V_{ij} e^{-y}$ (approximating the cap rate)

The basic CRR binomial model: Extension 3 Adjustments to existing improved property price – Dividends (continued)

- Note: As with depreciation, dividend payments are paid on the *whole* property, not simply the improvements. One component, r, represents the return *on* capital to the property, while a second component, δ , represents the return *of* capital from the property, which represents depreciation, the reduction of value of the property. A third component λ may also be present. $y = r + \delta + \lambda$

- If $y \equiv r + \delta$, the investors are compensated exactly for the amount of depreciation that reduced the value of the improved property and the return on capital to the property. However, if $\lambda > 0$, *excess distribution* exists that further reduces the value of the improved property beyond depreciation. Thus, the relative magnitudes of y, λ, and δ are important in influencing the value of the redevelopment option

- Caveat: δ *a*ssumed constant over all time and state space. Realistic from generalized *ex ante* perspective, but may not reflect reality in certain situations. Possible dynamic extensions to estimation?

The extended CRR binomial model: Modified parameters for existing improved property assuming stochastic construction costs, depreciation, and dividends

- $V_{ij} - K \quad \rightarrow \quad V_{ij} / K_{ij} - 1$

- $\sigma_V{}^2 \quad \rightarrow \quad \sigma_{V/K}{}^2 = \sigma_V{}^2 + \sigma_K{}^2 - 2\, \rho(V, K)\, \sigma_V\, \sigma_K$

- $V_{i\,j+1} = u\, V_{ij}$ and $V_{i+1\,j+1} = d\, V_{ij}$, where $u = e^{\sigma}$ and $d = e^{-\sigma} \quad \rightarrow$
  $(V'_{i\,j+1}/K'_{ij+1}) = u'\,(V'_{i+1j}/K'_{i+1j})$ and $(V'_{i+1\,j+1}/K'_{i+1j+1}) = d'\,(V'_{i+1j}/K'_{i+1j})$, where
  $u' = e^{(\sigma V/K - y - \delta)}$ and $d' = e^{(-\sigma V/K - y - \delta)}$

- $p = (e^r - d)/(u - d) = (e^r - e^{-\sigma})/(e^{\sigma} - e^{-\sigma}) \quad \rightarrow$
  $p' = (e^r - d')/(u' - d') = (e^r - e^{-(\sigma V/K - y - \delta)})/(e^{(\sigma V/K - y - \delta)} - e^{-(\sigma V/K - y - \delta)})$

The basic CRR binomial model: Extension 4
Detailing the structure of redevelopment costs and specifying the redevelopment option

- Our basic model did not consider the proper formulation of the *redevelopment* option.

- We specify the redevelopment option as an American call option, as we did the development option.

- The difference is the potential redeveloper is not working off of stochastic expectations of future property prices and construction costs, but the stochastic expectation of future property prices and construction costs *plus other transaction costs* of the conversion, which are also stochastic and may be significant.

- We consider the following three transaction costs (K_R) :
  - The cost to build a new property (K_N)
  - The cost to demolish the existing property (αV_{BE})
  - The opportunity cost of foregone returns from the existing property (V_E)

The final extended CRR binomial model: Adding consideration of transaction costs related to redevelopment

- $V_{ij} / K_{ij} - 1 \rightarrow V_{Nij} / K_{Rij} - 1 \rightarrow V_{Nij} / (K_{Nij} + V_{Eij} + \alpha V_{BEij}) - 1$
- $\sigma_{V/K}^2 = \sigma_V^2 + \sigma_K^2 - 2\,\rho(V, K)\,\sigma_V\,\sigma_K \rightarrow \sigma_{VN/KR}^2 = \sigma_{VN}^2 + \sigma_{KR}^2 - 2\,\rho(V_N, K_R)\,\sigma_{VN}\,\sigma_{KR}$
- $(V'_{i\,j+1}/K'_{ij+1}) = u'\,(V'_{i+1j}/K'_{i+1j})$ and $(V'_{i+1\,j+1}/K'_{i+1j+1}) = d'\,(V'_{i+1j}/K'_{i+1j})$, where $u' = e^{(\sigma V/K - y - \delta)}$ and $d' = e^{(-\sigma V/K - y - \delta)} \rightarrow$

  $(V'_{Ni\,j+1}/K'_{Rij+1}) = u''(V'_{Ni+1j}/K'_{Ri+1j})$ and $(V'_{Ni+1\,j+1}/K'_{Ri+1j+1}) = d''(V'_{Ni+1j}/K'_{Ri+1j})$, where $u'' = e^{(\sigma VN/KR - y - \delta)}$ and $d'' = e^{(-\sigma VN/KR - y - \delta)}$

- $p' = (e^r - d')/(u' - d') = (e^r - e^{-(\sigma V/K - y - \delta)})/(e^{(\sigma V/K - y - \delta)} - e^{-(\sigma V/K - y - \delta)}) \rightarrow$

  $p'' = (e^r - d'')/(u'' - d'') = (e^r - e^{-(\sigma VN/KR - y - \delta)})/(e^{(\sigma VN/KR - y - \delta)} - e^{-(\sigma VN/KR - y - \delta)})$

- Thus $V_{Nij} / K_{Nij} - 1 \rightarrow V_{Nij} / (K_{Nij} + V_{Eij} + \alpha V_{BEij}) - 1 \rightarrow$

  $V_{0N}\, e^{\sigma VN - yN} / [K_{0N}\, e^{(\sigma KN - yKN)} + V_{0E}\, e^{(\sigma VE - yE - \delta E)} + \alpha K_{0BE}\, e^{(\sigma KE - yE - \delta BE)}] - 1$

Simulation I: The extended CRR binomial model for the Land Development Option

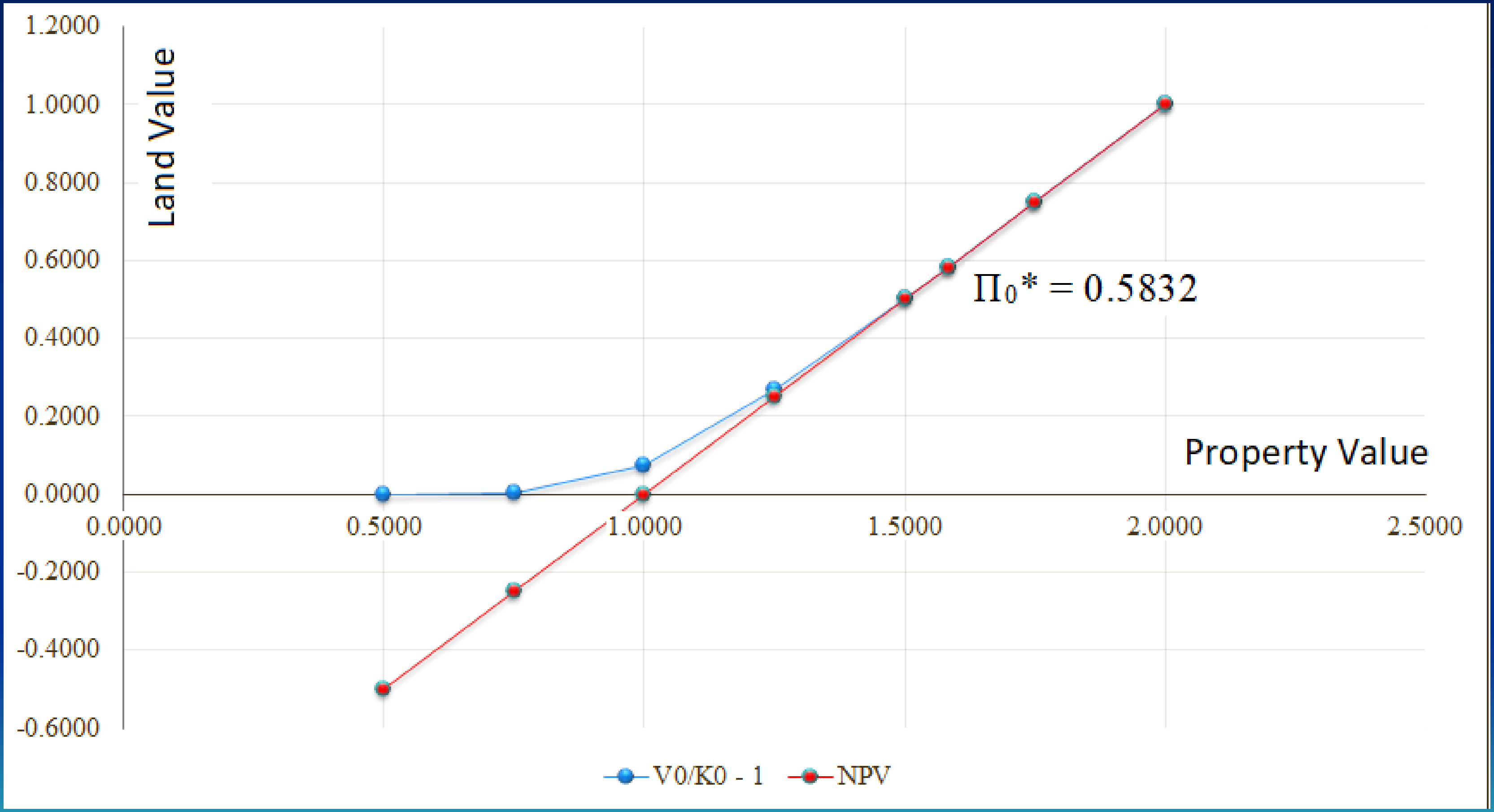
INPUT ASSUMPTIONS:

• Risk-free rate (r)	1.00%
• Property cash dividend rate (y)	6.00%
• Contractor dividend rate (y_C)	2.00%
• Initial property price (V_0)	0.5000 – 2.0000
• Initial development cost (K_0)	1.0000
• Correlation coefficient between price & cost ($\rho_{V/K}$)	0.8000
• Property volatility (σ_V)	5.00 - 20.00%
• Cost volatility (σ_K)	10.00%

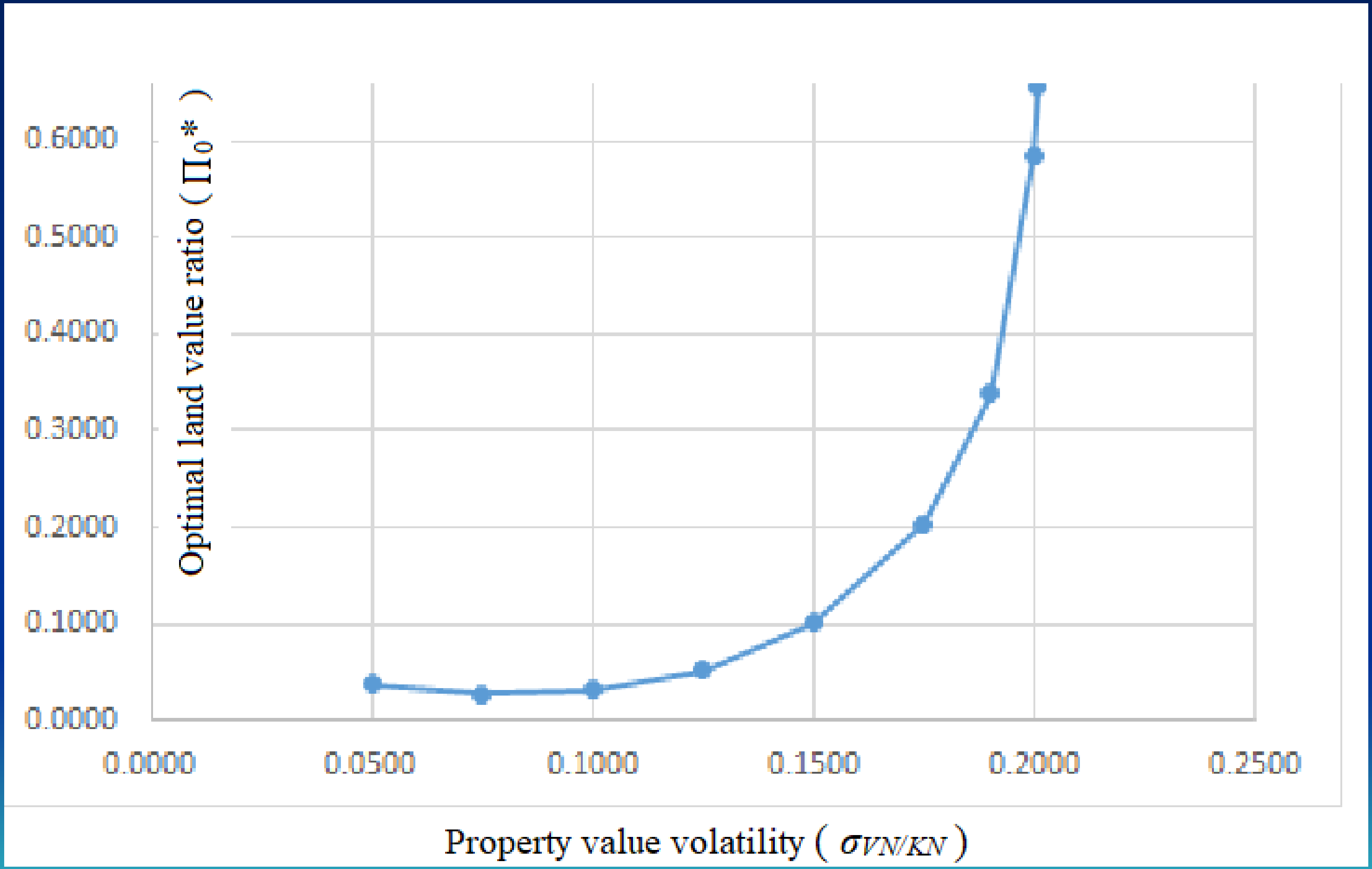
OUTPUT:

1. $\Pi_0 = (V_0' / K_0 - 1)$ and Π_0 for $0.50 \leq V_0 \leq 2.00$ **[Note: $\Pi_0 = \Pi_0^{max} = .049$ at $V_0' / K = 1.00$]**
2. $\Pi_0 = (V_0' / K_0 - 1)$ and Π_0^* for $0.05 \leq \sigma_V \leq 0.20$

Simulation results: The land development option value ($\Pi_0 = V_0' / K_0 - 1$) vs *NPV* by property value (V_0 / K_0)



Simulation results: The optimal land value ratio ($\Pi_0^* = V_0'^*/K_0 - 1$) by property value volatility (σ_V)



Simulation II: The extended CRR binomial model for the Land Redevelopment Option

- From point of view of a redeveloper deciding when redevelopment of a property that has just been built (at HBU) on the subject site will be "ripe" for redevelopment, and the current value of his redevelopment option.
- In base case, we assume "new" property (*N*) will have identical characteristics to recently built property (i.e., same parameters as "existing" property (*E*)), with the exception of cost of redevelopment ($K_{ij} \rightarrow K_{Rij} = K_{Nij} + V_{Eij} + \alpha V_{BEij}$)
- Existing (recently built) property assumed to experience depreciation at rate δ after development and $\sigma_{V/K} \rightarrow \sigma_{VN/KR}$.
- Correlation between V_N and K_R derived from correlations among components
- Results of interest: (1) Redevelopment option value at time of development of existing property (including comparison to development option value) and (2) timing of future redevelopment, depending upon state space over time

Simulation II: The extended CRR binomial model for the Land Redevelopment Option (continued)

ADDITIONAL INPUT ASSUMPTIONS FOR EXISTING PROPERTY:

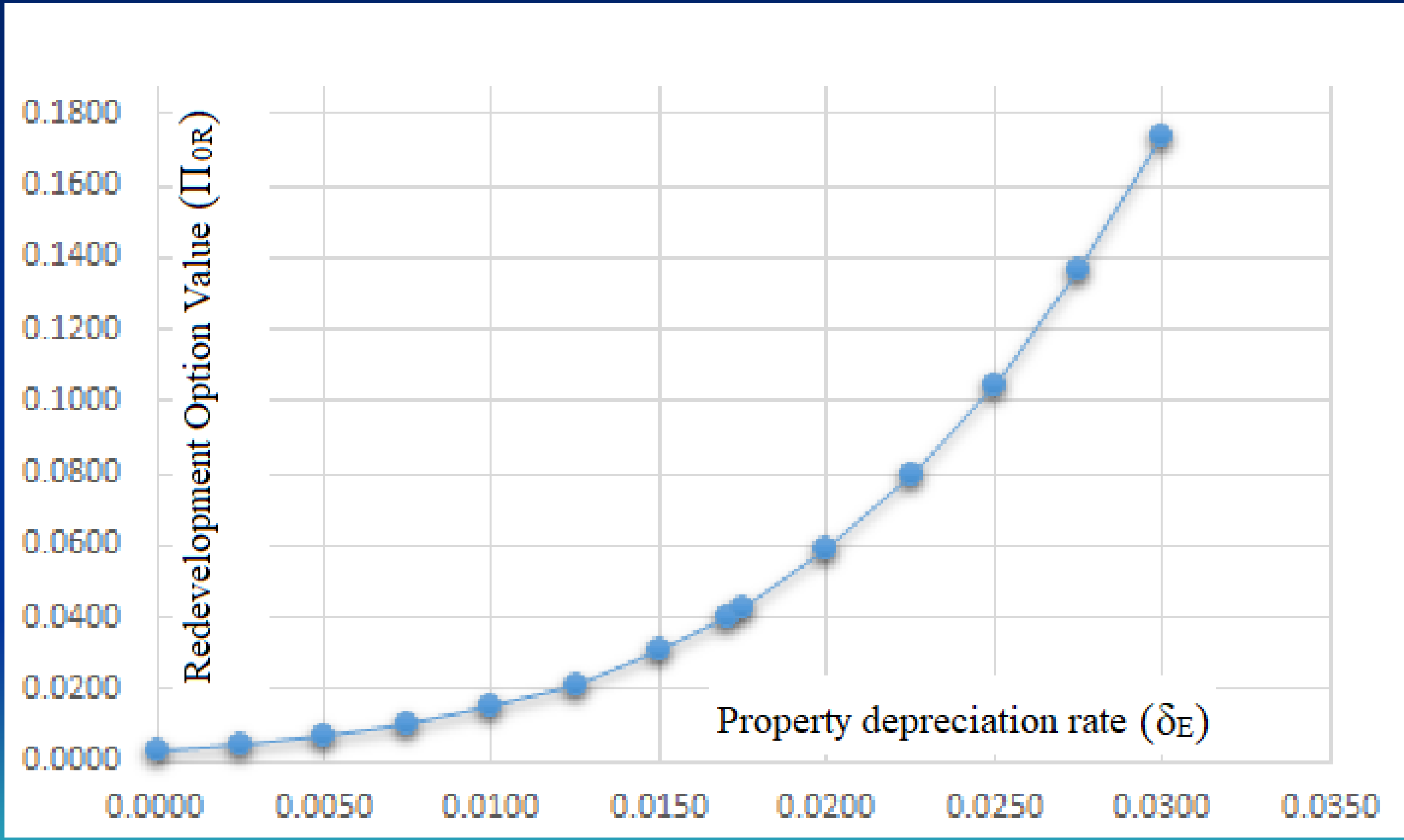
- Existing Development Net Demolition Cost as % of V_{BE} (α) 8.00%
- Existing Development Rate of Depreciation as % of V_E (δ_E) 1.70%
- Existing Development Rate of Depreciation as % of V_{BE} (δ_{BE}) 2.69% (derived)
- Existing Development Initial Value (V_{0E}) 1.5832
- Existing Development Initial Cost ($K_{0E} = V_{0E}$) 1.0000
- Additional assumed volatilities for: σ_{VE} , σ_{VBE}
- Additional assumed correlation coefficients for: ρ_{VNVE} , ρ_{VNVBE} , ρ_{KNVE} , ρ_{KNVBE} , ρ_{VEVBE}
- Derived estimates of: $\sigma_{VN/KN}$, σ_{KR} , $\sigma_{VN/KR}$

OUTPUT:

1. $\Pi_{0R} = (V_{0N}^* / K_{0R} - 1)$ for $0.00 \leq \delta_E \leq 3.00\%$

Note: $\Pi_{0R} = .040$ at $\delta_E = 1.70\%$. Comparable to $\Pi_0^{max} = .049$ for new development

Simulation results: The land redevelopment option value at time of initial site development ($\Pi_{0R} = V_{0E}^*/K_{0R} - 1$) by existing property depreciation rate (δ_E)



Simulation results: Optimal redevelopment timing by state space ((i^*, j^*) s.t. $\Pi_{i^*j^*R}{}^* = V_{i^*j^*R}{}^*/K_{i^*j^*R}{}^* - 1$) [$\Pi_{0R} = .0398$ and $\delta_E = 1.70\%$]

YEAR	0	1	2	3	4	5	6	7	8	9	10	11	12
ST OF NATURE													
0	0.0398	0.0491	0.0604	0.0737	0.0894	0.1079	0.1293	0.1541	0.1824	0.2147	0.2512	0.2921	0.3378
1		0.0285	0.0356	0.0444	0.0549	0.0675	0.0824	0.1000	0.1206	0.1445	0.1721	0.2036	0.2393
2			0.0198	0.0251	0.0317	0.0397	0.0495	0.0613	0.0755	0.0922	0.1120	0.1350	0.1617
3				0.0133	0.0171	0.0219	0.0279	0.0353	0.0443	0.0553	0.0686	0.0845	0.1033
4					0.0086	0.0112	0.0146	0.0189	0.0243	0.0310	0.0393	0.0495	0.0619
5						0.0053	0.0071	0.0094	0.0123	0.0161	0.0209	0.0269	0.0345
6							0.0031	0.0043	0.0057	0.0077	0.0102	0.0135	0.0177
7								0.0018	0.0024	0.0033	0.0046	0.0062	0.0083
8									0.0009	0.0013	0.0018	0.0026	0.0035
9										0.0005	0.0007	0.0009	0.0013
10											0.0002	0.0003	0.0005
11												0.0001	0.0001
12													0.0000

YEAR	13	14	15	16	17	18	19	20	21	22	23	24	25
ST OF NATURE													
0	0.3884	0.4442	0.5054	0.5723	0.6450	0.7238	0.8091	0.9011	1.0005	1.1076	1.2231	1.3478	1.4823
1	0.2796	0.3246	0.3747	0.4300	0.4907	0.5571	0.6293	0.7077	0.7924	0.8839	0.9826	1.0889	1.2035
2	0.1924	0.2273	0.2669	0.3113	0.3607	0.4155	0.4758	0.5417	0.6135	0.6914	0.7757	0.8667	0.9647
3	0.1254	0.1512	0.1810	0.2152	0.2539	0.2977	0.3465	0.4008	0.4606	0.5261	0.5975	0.6750	0.7589
4	0.0768	0.0947	0.1159	0.1407	0.1695	0.2028	0.2408	0.2838	0.3321	0.3858	0.4451	0.5102	0.5813
5	0.0438	0.0553	0.0693	0.0861	0.1063	0.1301	0.1580	0.1903	0.2274	0.2697	0.3173	0.3705	0.4294
6	0.0231	0.0299	0.0384	0.0489	0.0619	0.0777	0.0967	0.1194	0.1463	0.1776	0.2138	0.2553	0.3022
7	0.0111	0.0148	0.0195	0.0255	0.0331	0.0427	0.0547	0.0693	0.0872	0.1088	0.1345	0.1647	0.2000
8	0.0048	0.0066	0.0090	0.0121	0.0161	0.0214	0.0281	0.0368	0.0476	0.0611	0.0778	0.0981	0.1226
9	0.0019	0.0027	0.0037	0.0051	0.0070	0.0096	0.0130	0.0176	0.0235	0.0311	0.0408	0.0531	0.0685
10	0.0007	0.0009	0.0014	0.0019	0.0027	0.0038	0.0054	0.0075	0.0103	0.0141	0.0191	0.0257	0.0343
11	0.0002	0.0003	0.0004	0.0006	0.0009	0.0013	0.0019	0.0028	0.0039	0.0056	0.0078	0.0109	0.0151
12	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004	0.0006	0.0009	0.0013	0.0019	0.0028	0.0040	0.0057
13	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0004	0.0005	0.0008	0.0012	0.0018
14		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005
15			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
16				0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18						0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19							0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20								0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21									0.0000	0.0000	0.0000	0.0000	0.0000
22										0.0000	0.0000	0.0000	0.0000
23											0.0000	0.0000	0.0000
24												0.0000	0.0000
25													0.0000

YEAR	26	27	28	29	30	31	32	33	34	35	36	37	38
ST OF NATURE													
0	1.6277	1.7823	1.9460	2.1193	2.3028	2.4971	2.7028	2.9206	3.1513	3.3955	3.6540	3.9278	4.2177
1	1.3270	1.4603	1.6042	1.7574	1.9196	2.0914	2.2732	2.4658	2.6697	2.8855	3.1141	3.3561	3.6124
2	1.0702	1.1839	1.3064	1.4384	1.5809	1.7327	1.8935	2.0637	2.2439	2.4348	2.6368	2.8508	3.0773
3	0.8493	0.9468	1.0516	1.1644	1.2859	1.4167	1.5578	1.7083	1.8676	2.0363	2.2149	2.4040	2.6043
4	0.6585	0.7419	0.8319	0.9288	1.0330	1.1450	1.2655	1.3951	1.5349	1.6840	1.8419	2.0091	2.1861
5	0.4941	0.5649	0.6417	0.7248	0.8144	0.9108	1.0144	1.1256	1.2452	1.3738	1.5122	1.6600	1.8165
6	0.3548	0.4133	0.4777	0.5482	0.6248	0.7076	0.7968	0.8927	0.9957	1.1063	1.2250	1.3525	1.4897
7	0.2406	0.2868	0.3388	0.3969	0.4610	0.5312	0.6076	0.6901	0.7791	0.8746	0.9771	1.0870	1.2049
8	0.1517	0.1858	0.2255	0.2710	0.3225	0.3801	0.4439	0.5139	0.5901	0.6725	0.7612	0.8564	0.9584
9	0.0875	0.1106	0.1384	0.1714	0.2101	0.2548	0.3056	0.3629	0.4264	0.4963	0.5724	0.6547	0.7431
10	0.0453	0.0593	0.0769	0.0985	0.1250	0.1567	0.1943	0.2381	0.2884	0.3452	0.4086	0.4784	0.5544
11	0.0207	0.0281	0.0378	0.0503	0.0663	0.0864	0.1113	0.1417	0.1780	0.2209	0.2705	0.3270	0.3902
12	0.0082	0.0115	0.0161	0.0223	0.0307	0.0417	0.0560	0.0743	0.0975	0.1263	0.1613	0.2031	0.2521
13	0.0027	0.0040	0.0058	0.0084	0.0120	0.0170	0.0239	0.0333	0.0458	0.0623	0.0835	0.1105	0.1440
14	0.0007	0.0011	0.0017	0.0026	0.0038	0.0057	0.0084	0.0123	0.0178	0.0254	0.0360	0.0503	0.0694
15	0.0002	0.0002	0.0004	0.0006	0.0010	0.0015	0.0023	0.0035	0.0054	0.0082	0.0122	0.0181	0.0266
16	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0005	0.0007	0.0012	0.0019	0.0030	0.0048	0.0075
17	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0005	0.0008	0.0014
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29				0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
31						0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32							0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
33								0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
34									0.0000	0.0000	0.0000	0.0000	0.0000
35										0.0000	0.0000	0.0000	0.0000
36											0.0000	0.0000	0.0000
37												0.0000	0.0000
38													0.0000

YEAR	39	40	41	42	43	44	45	46	47	48	49	50
ST OF NATURE												
0	4.5247	4.8497	5.1938	5.5581	5.9439	6.3524	6.7849	7.2429	7.7278	8.2412	8.7849	9.3605
1	3.8837	4.1710	4.4752	4.7973	5.1383	5.4994	5.8818	6.2866	6.7153	7.1691	7.6497	8.1585
2	3.3171	3.5711	3.8400	4.1247	4.4262	4.7454	5.0834	5.4413	5.8202	6.2214	6.6462	7.0960
3	2.8163	3.0408	3.2785	3.5302	3.7967	4.0789	4.3776	4.6940	5.0289	5.3836	5.7591	6.1568
4	2.3735	2.5720	2.7821	3.0046	3.2402	3.4896	3.7538	4.0334	4.3295	4.6430	4.9750	5.3265
5	1.9822	2.1576	2.3434	2.5400	2.7483	2.9688	3.2022	3.4495	3.7112	3.9883	4.2818	4.5925
6	1.6362	1.7913	1.9555	2.1293	2.3134	2.5083	2.7147	2.9333	3.1646	3.4096	3.6690	3.9437
7	1.3314	1.4674	1.6126	1.7663	1.9290	2.1013	2.2838	2.4769	2.6815	2.8980	3.1274	3.3702
8	1.0677	1.1848	1.3105	1.4454	1.5892	1.7415	1.9028	2.0736	2.2544	2.4458	2.6485	2.8632
9	0.8380	0.9396	1.0484	1.1648	1.2896	1.4235	1.5660	1.7170	1.8768	2.0461	2.2252	2.4150
10	0.6366	0.7249	0.8195	0.9208	1.0290	1.1449	1.2689	1.4018	1.5431	1.6927	1.8511	2.0188
11	0.4600	0.5361	0.6183	0.7065	0.8008	0.9018	1.0097	1.1250	1.2483	1.3803	1.5203	1.6686
12	0.3082	0.3715	0.4414	0.5176	0.5997	0.6878	0.7819	0.8826	0.9902	1.1051	1.2279	1.3590
13	0.1846	0.2329	0.2889	0.3522	0.4224	0.4987	0.5809	0.6687	0.7627	0.8632	0.9706	1.0853
14	0.0942	0.1259	0.1653	0.2129	0.2689	0.3326	0.4032	0.4797	0.5616	0.6493	0.7431	0.8434
15	0.0386	0.0551	0.0775	0.1070	0.1449	0.1920	0.2482	0.3127	0.3839	0.4603	0.5420	0.6295
16	0.0117	0.0179	0.0272	0.0407	0.0600	0.0869	0.1231	0.1698	0.2272	0.2932	0.3643	0.4405
17	0.0023	0.0038	0.0062	0.0102	0.0164	0.0264	0.0417	0.0650	0.0990	0.1463	0.2072	0.2734
18	0.0002	0.0004	0.0007	0.0012	0.0022	0.0039	0.0069	0.0124	0.0221	0.0395	0.0704	0.1256
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
28	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
29	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
31	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
32	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
33	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
36	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
37	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
38	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
39	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
40		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
41			0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
42				0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
43					0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
44						0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
45							0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
46								0.0000	0.0000	0.0000	0.0000	0.0000
47									0.0000	0.0000	0.0000	0.0000
48										0.0000	0.0000	0.0000
49											0.0000	0.0000
50												0.0000

Results and implications from the CRR models and simulations:

1. THOSE THINGS DONE:
   - Demonstrated the usefulness of the CRR model, appropriately modified, to extract realistic estimates of both the development and redevelopment option value for land under different scenarios
   - Capable also of providing guidance for developers, investors, planners, assessors decision making: e.g. anticipated timing of activity, density and nature of development/redevelopment
   - Does not require an inordinate number of input parameters, or parameters that are difficult or impossible to obtain
   - Provides estimates that can be validated using available data and analysis
   - Demonstrated the differing degree of sensitivity of results to magnitudes of input parameters
   - Demonstrated high sensitivity of results to correlations between and among volatilities of certain input parameters
   - Theoretically, and assuming parameters of reasonable magnitudes, capable of being applied to a single site at a specific point in time
   - Also theoretically able to evaluate compound options of development and redevelopment simultaneously at and for different sites at different points in time

Results and implications from the CRR models and simulations (continued):

1. THOSE THINGS LEFT UNDONE:

   – Completion of parameter estimation using Maricopa County data and other sources

   – Completion of model calibration and validation/correlation with parameter estimates

   – Residential and residential land hedonic estimation at time of development and redevelopment, when intrinsic land value, without embedded land option value, theoretically can be extracted.

   – Undertake additional programming to merge development and redevelopment land option values, update equilibrium land value ratios at time of development

   – Additional simulations using site-specific parameters to evaluate sensitivity of land development/ redevelopment option values

   – Modify static assumptions about evolution of σ_V over time and state space. Dynamic models, such as that developed by Unison Investment Management (*Robust Home Price, Return and Volatility Indices*, 2019), offer promise

   – Extension of empirical analysis to multifamily and commercial markets

   – Integrate all components of models into single whole and create a straightforward dashboard that is user-friendly for professional applications. Look into using existing off-the-shelf option pricing models as templates for this task